Self-organization Using Synaptic Plasticity

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Abstract

- **Neural activity self-regulates** to prevent neural circuits from becoming hyper- or hypoactive by means of homeostatic processes [9].
- **Optimal information processing** in complex systems is attained at a critical point, near a transition between an ordered and an unordered regime of dynamics [5, 3, 8, 6].
- **Self-Organized Criticality** (SOC) [1, 2] has been proposed as a mechanism for neural systems which evolve *naturally* to a critical state without external tuning.
- Regulation mechanism may be provided by **synaptic plasticity**, as proposed in [7].

In this work we **analytically derive a local synaptic rule** that can drive and maintain a neural network near the critical state. According to the proposed rule, synapses are either strengthened or weakened whenever a post-synaptic neuron receives either more or less input from the population than the required to fire at its *natural* frequency. This simple principle is enough for the network to **self-organize at a critical region where the dynamic range is maximized**. We illustrate this using a model of non-leaky spiking neurons with delayed coupling.

The model : Nonleaky integrate-and-fire model

- Activation state a_i of a neuron i evolves toward a threshold L. When L is reached, a spike is propagated to other neurons.
- When i receives a spike, a_i is increased according to the synaptic efficacy ϵ_{ij}.
- Subthreshold (discrete) dynamics of neuron i, $i = \{1..N\}$:

$$a_i(t+1) - a_i(t) = \Delta I_{noise}(t) + I_{rec}(t - t_{delay})$$

I_{noise} **stochastic process** $\rightarrow$ Bernoulli process with noise rate p

$I_{rec}(t)$ **population induced activity** $\rightarrow$ $\sum_{j=1, j\neq i}^{N} \epsilon_{ij} H_L(a_j(t))$

t_{delay} **propagation delay** $\rightarrow$ 1

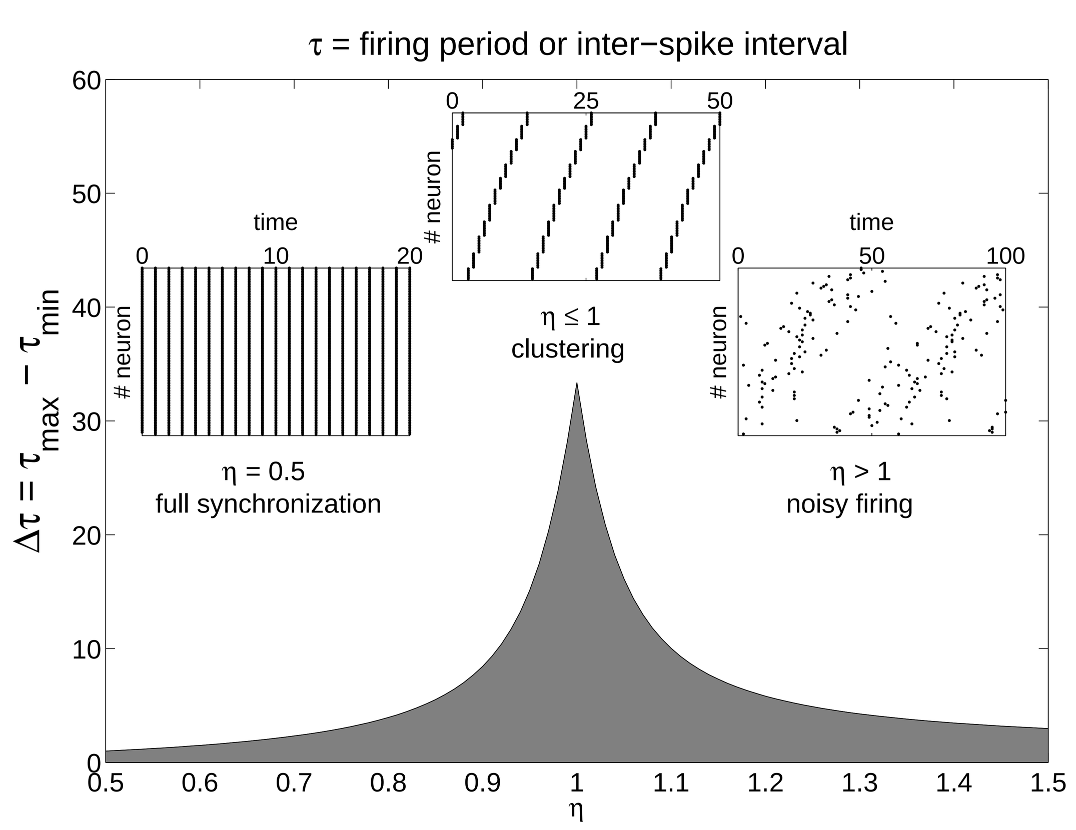
$H_L(x)$ is the Heaviside step function: $H_L(x) = 1$ if $x \geq L$, and 0 otherwise.

Model with 'Static' Synapses

- Degree of interaction between the units ($\langle\epsilon\rangle \equiv$ mean synaptic efficacy) :

$$\eta = \frac{L-1}{(N-1)\langle\epsilon\rangle} \qquad \begin{cases} \textbf{Sub-critical} & \text{for } \eta > 1 \\ \textbf{Critical} & \text{for } \eta = 1 \\ \textbf{Super-critical} & \text{for } \eta < 1 \end{cases}$$

- Transition from irregular, noise-driven, dynamics to regular, self-sustained behavior at a critical coupling strength $\eta = 1$.



- Dynamic range $\Delta\tau = \tau_{max} - \tau_{min}$ is maximized at $\eta = 1$ [4].

Synaptic plasticity causing SOC

The Dissipated Spontaneous Activity

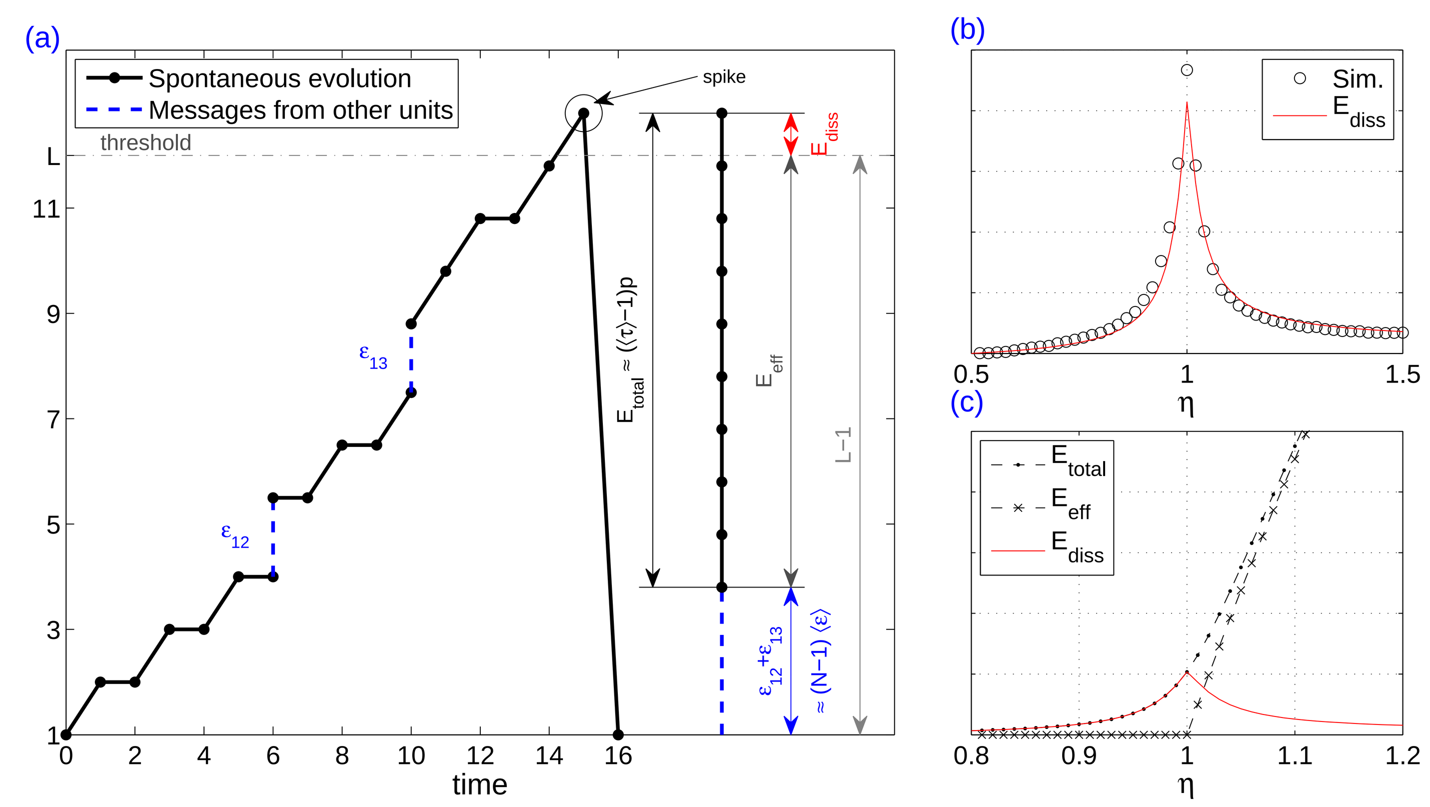
We define average magnitudes during the period τ of a neuron:

- Average **total spontaneous evolution**: $E_{total} = (\langle\tau\rangle - 1)p$.
- Average effective spontaneous evolution:

$$E_{eff} = \max\{0, L - 1 - (N-1)\langle\epsilon\rangle\}$$

➜ Their subtraction gives the dissipated spontaneous evolution:

$$E_{diss} = E_{total} - E_{eff}.$$



(a): Example of temporal evolution of $a_i(t)$ during a period of length $\tau = 15$.
(b): Empirical versus analytical E_{diss} (E_{diss} is maximized at $\eta = 1$).
(c): Analytical curves of E_{total}, E_{eff} and E_{diss} around the critical point.

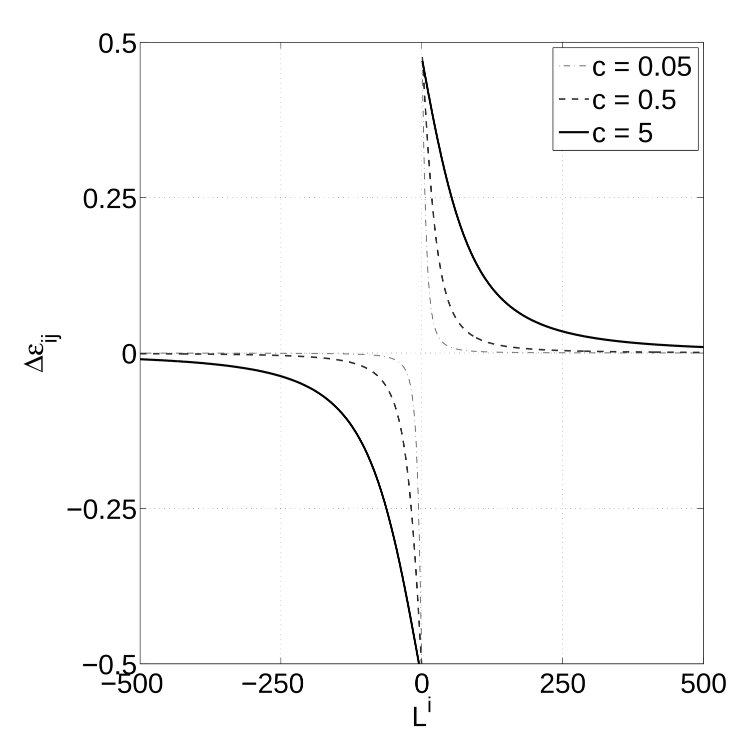
Local Plasticity Rule

Individual synapses ϵ_{ij} are updated in the direction of the gradient of E_{diss} each time a post-synaptic neuron i fires:

$$\Delta\epsilon_{ij} = \kappa\frac{\partial E^i_{diss}}{\partial \epsilon_{ij}} = \kappa\left(\frac{-L^i - c}{2\sqrt{(L^i + 2c)^2 + 2c(L - L^i)}} + \frac{\mathrm{sgn}(L^i)}{2}\right)$$

- L^i: **Effective threshold of post-synaptic neuron** i
  Difference between the threshold L and the activity received by neuron i from the population in the last period.
- κ, c are arbitrary constants (can be different for every synapse).

$$\Delta\epsilon_{ij} = \begin{cases} \textbf{Strenghtening } (>0) & \text{for } L^i > 0 \\ \textbf{Not defined} & \text{for } L^i = 0 \\ \textbf{Weakening } (<0) & \text{for } L^i < 0 \end{cases}$$

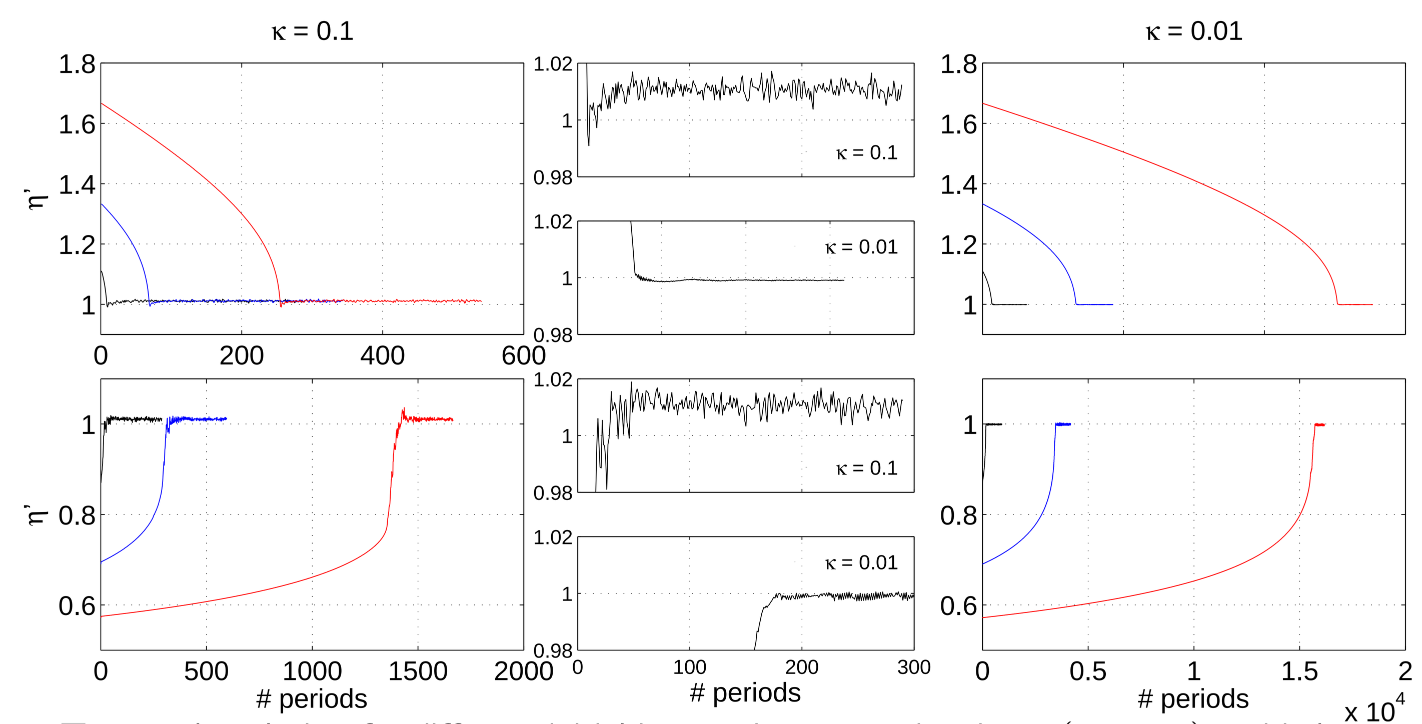


➜ The resulting plasticity rule involves only **local terms**.

Simulations

Model with 'Plastic' Synapses

- **Setup**: $N = 500$, $L = 500$, $p = 0.9$. Synapses initialized homogeneously.



Temporal evolution for different initial interaction strengths above (top row) and below (bottom row) the critical point for different values of κ (left and right).

- After an initial transient, the network converges to a critical regime, where the dynamics balances between a predictable pattern of activity and uncorrelated random behavior.
- κ determines the speed of convergence and the quality and stability of the dynamics at the critical state.
- Analytical approximations for time of convergence given in the paper.

Conclusions

- We have derived a local synaptic mechanism that induces global homeostasis towards an optimal dynamic state.
- The proposed synaptic rule generalizes SOC rule proposed in [3] for binary neurons to the case of spiking neurons.
- Results indicate that effects of fluctuations due to noise are minimized at the critical state ($\eta = 1$).

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