

# Introduction to Spectral Processing

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# Spectral Representation

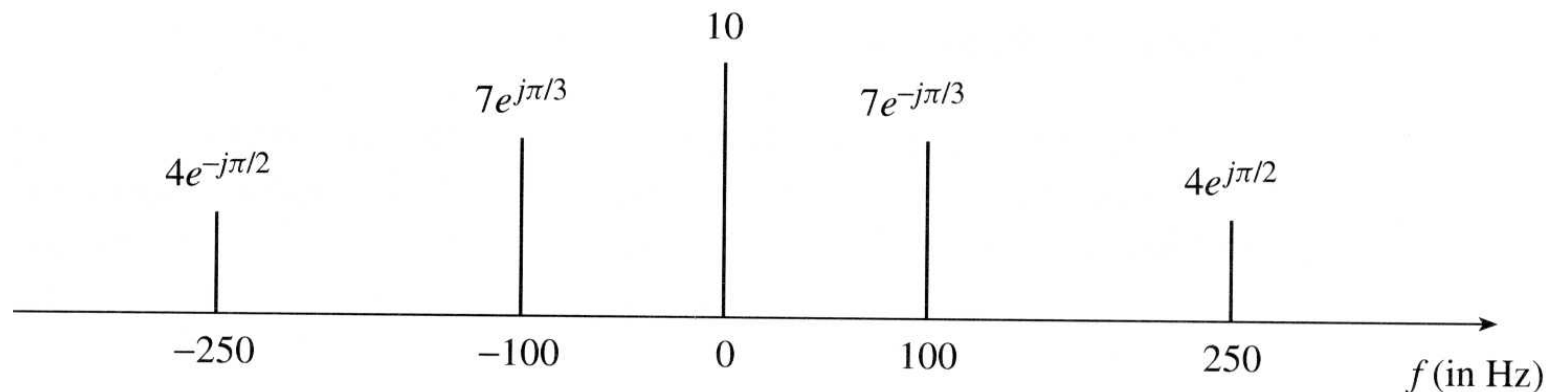
$$\begin{aligned}x(t) &= A_0 + \sum_{k=1}^N A_k \cos(2\pi f_k t + \phi_k) \\&= A_0 + \sum_{k=1}^N \operatorname{Re}\left\{A_k e^{j(2\pi f_k t + \phi_k)}\right\} \\&= A_0 + \operatorname{Re}\left\{\sum_{k=1}^N A_k e^{j\phi_k} e^{j2\pi f_k t}\right\} \\&= X_0 + \sum_{k=1}^N \left\{\frac{X_k}{2} e^{j2\pi f_k t} + \frac{X_k^*}{2} e^{-j2\pi f_k t}\right\}\end{aligned}$$

where  $X_k = A_k e^{j\phi_k}$

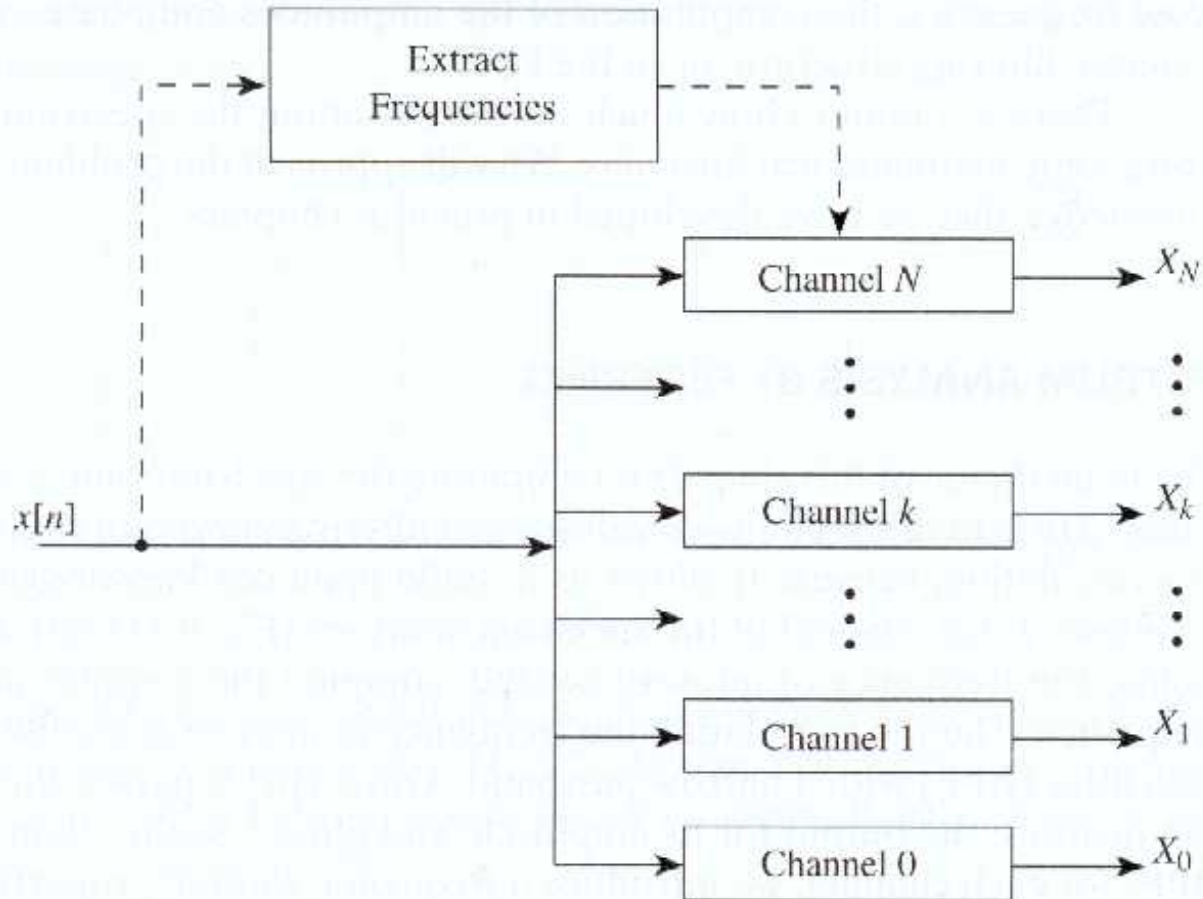
A spectrum is a sequence of pairs of numbers:

$$\left\{ (\mathbf{X}_0, 0), \left( \frac{1}{2} \mathbf{X}_1, f_1 \right), \left( \frac{1}{2} \mathbf{X}_1^*, -f_1 \right), \left( \frac{1}{2} \mathbf{X}_2, f_2 \right), \left( \frac{1}{2} \mathbf{X}_2^*, -f_2 \right), \dots \right\}$$

The spectrum of:  $x(t) = 10 + 14 \cos\left(200\pi t - \frac{\pi}{3}\right) + 8 \cos\left(500\pi t + \frac{\pi}{2}\right)$



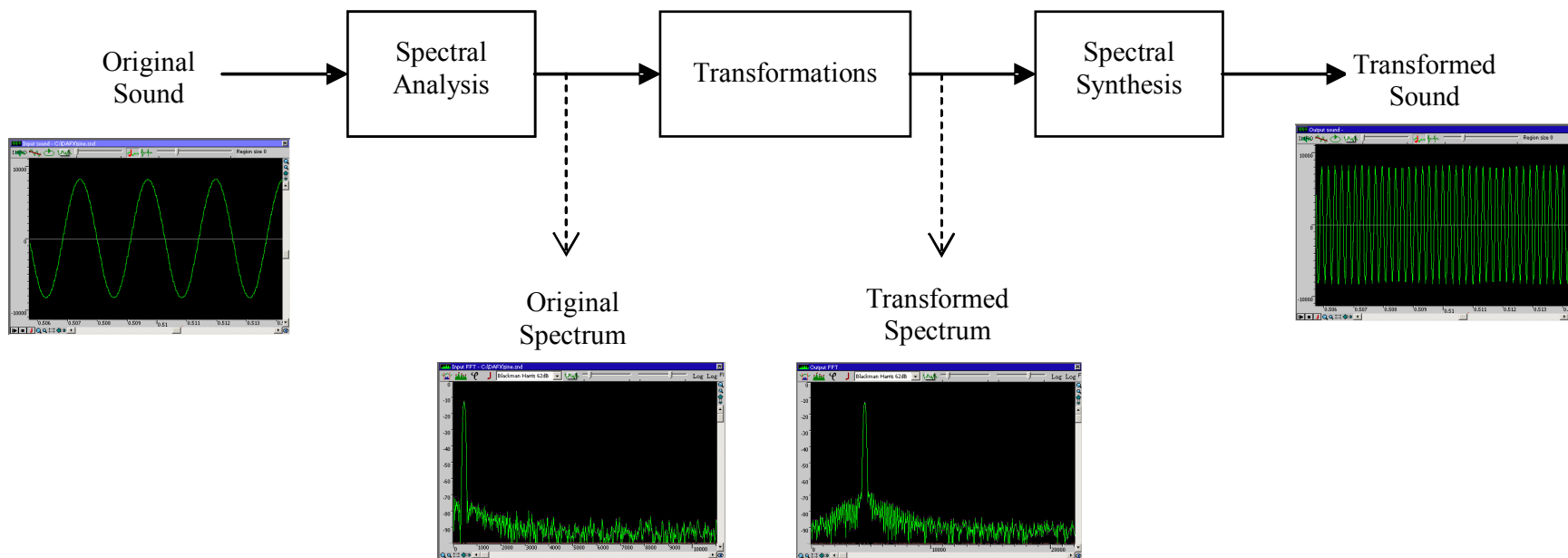
## Spectral analysis:



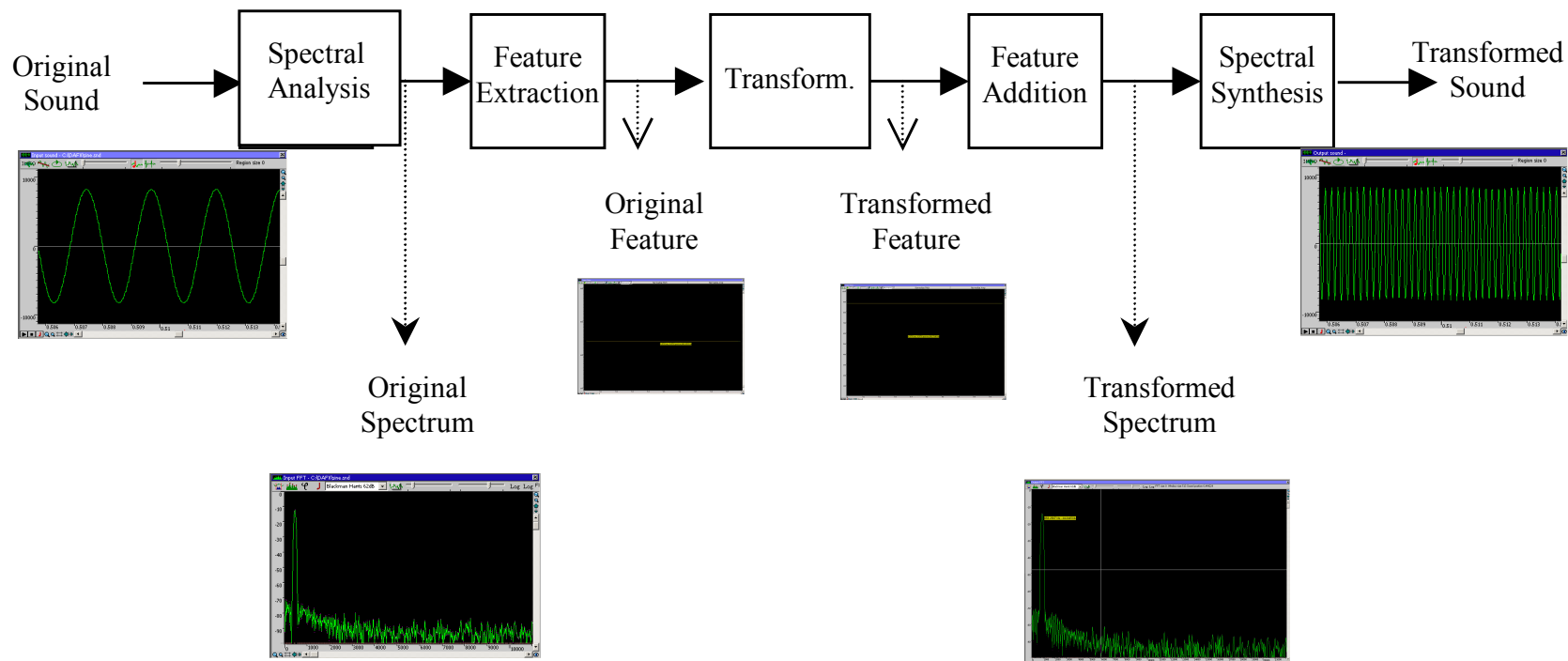
# Historical background

- Additive synthesis (1970s)
- Heterodyne filter (Moorer 1973)
- Phase Vocoder (Moorer 1978)
- Sinusoidal models (McAulay & Quatieri 1984)
- PARSHL (Smith & Serra 1987)
- SMS (Serra 1989)
- Other implementations: SVP, Lemur, CAST, ...

# Spectral processing framework



# Higher-level spectral processing:



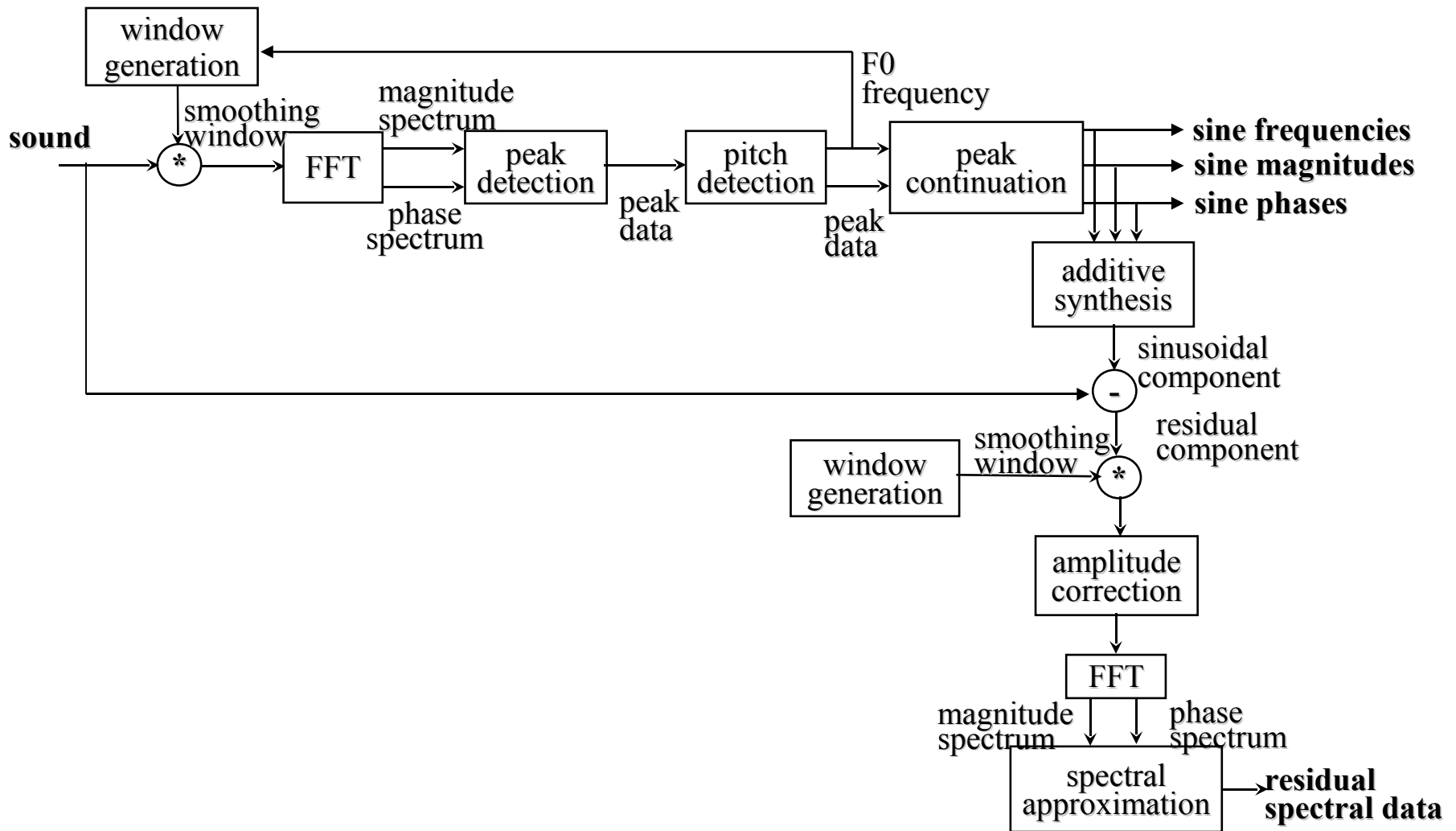
# Complementary spectral models

- Short-Time Fourier Transform (STFT)
- Sinusoidal Model
- Sinusoidal plus Residual Model
- Higher Level Attributes

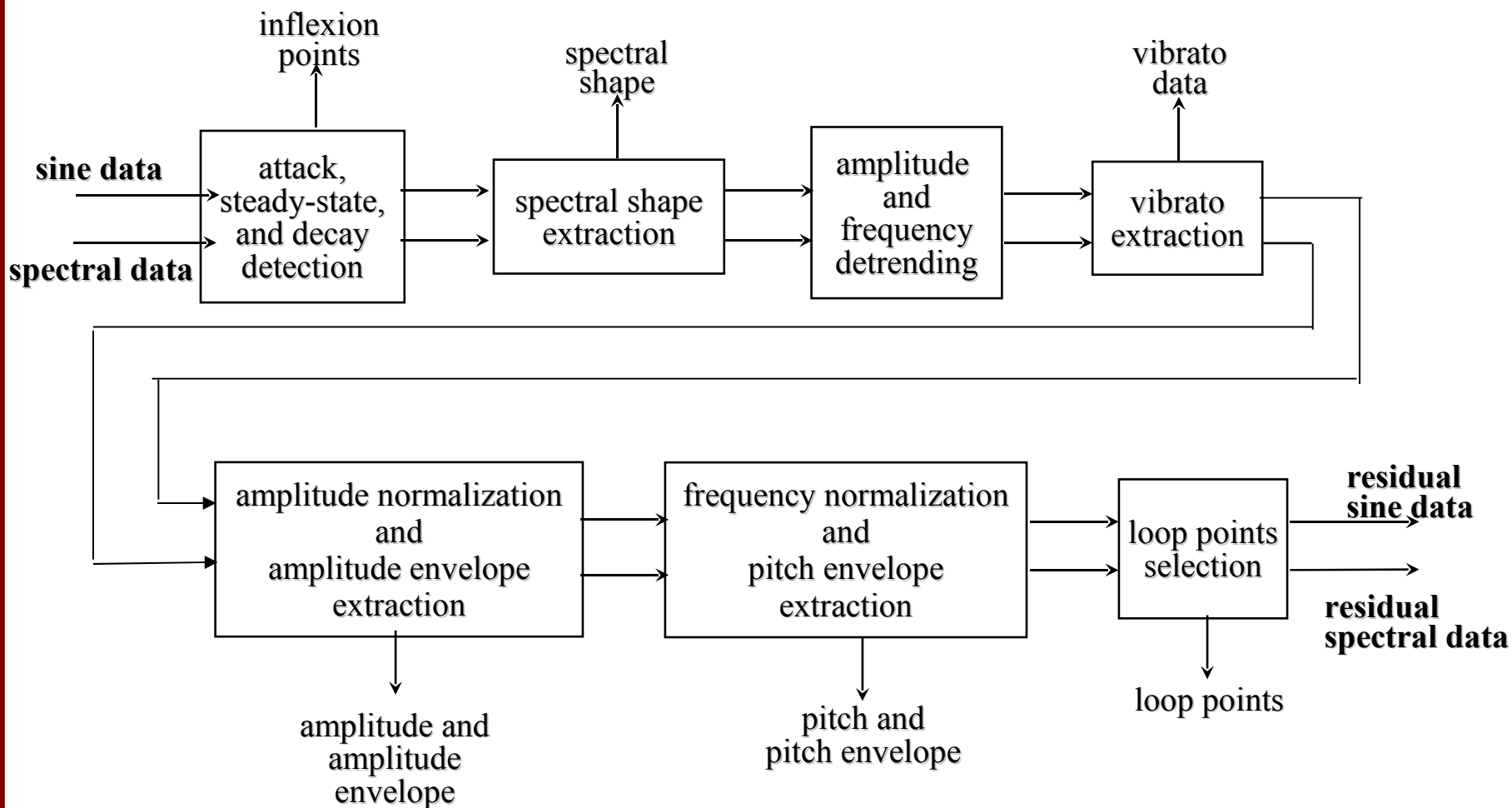
# Sound synthesis compromises

- Sound fidelity
- Flexibility
- Generality
  
- Memory consumption
- Compute time

# Spectral analysis



# Higher level Attributes



# Spectral synthesis

