Planning Graphs and Knowledge Compilation

Héctor Geffner
ICREA and Universitat Pompeu Fabra
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Planning as SAT (Kautz and Selman)

- **Encode:** Map Strips problem $P$ with horizon $n$ into a propositional theory $T$

- **Solve:** Using a SAT solver, determine if $T$ is consistent, and if so, find a model

- **Decode:** Extract plan from model
Our goal

Use of propositional logic for **defining and computing lower bounds for planning** (admissible heuristics)

- understand the **planning graph** construction as a **precise form of inference**
- exploit account to uncover **relations** (e.g., to variable elimination) and introduce **generalizations** (e.g., incomplete information)
A problem in Strips is a tuple \( \langle A, O, I, G \rangle \) where

- \( A \) stands for set of all atoms (boolean vars)
- \( O \) stands for set of all operators (ground actions)
- \( I \subseteq A \) stands for initial situation
- \( G \subseteq A \) stands for goal situation

The operators \( o \in O \) represented by three lists

--- the Add list \( Add(o) \subseteq A \)
--- the Delete list \( Del(o) \subseteq A \)
--- the Precondition list \( Pre(o) \subseteq A \)

The task is to find a plan: a sequence of applicable actions that maps \( I \) into \( G \) . . .
Lower Bounds and Planning Graphs

• Build graph with layers $P_0$, $A_0$, $P_1$, $A_1$, . . . where

\[
P_0 = \{ p \in s \}
\]
\[
A_i = \{ a \in O \mid Prec(a) \subseteq P_i \}
\]
\[
P_{i+1} = \{ p \in Add(a) \mid a \in A_i \}
\]

• Graph represents lower bound for achieving $G$ from $s$:

\[
h_{\text{max}}(s) = \min i \text{ such that } G \subseteq P_i
\]

Need No-op($p$) action for each $p$: $Prec = Add = \{p\}$
More Informed $h$ in Graphplan

- Planning graph in Graphplan also keeps track of pairs that cannot be reached simultaneously in $i$ steps, $i = 0, 1, \ldots$
  - action pair mutex at $i$ if incompatible or preconditions mutex at $i$
  - atom pair mutex at $i+1$ if supporting action pairs all mutex at $i$

- Mutexes computed along with planning graph and yield more informed admissible $h$

$$h_G(s) \overset{\text{def}}{=} \min i \text{ s.t. } G \subseteq P_i \text{ and } G \text{ not mutex at } i$$

Graphplan is an IDA* regression solver driven by this heuristic
Lower Bounds crucial in Planning and Problem Solving

• LBs explain performance gap between Graphplan and predecessors

• In SAT/CSP planning models, LBs represent implicit constraints that speed up the search:

  SAT/CSP approaches to planning indeed do not encode the planning problem directly but its planning graph

• Our main goal in this work: understand derivation of these LBs or implicit constraints in the planning graph as a precise form of inference
Consider following heuristic $h$ where $T$ encodes Strips problem with horizon $n$ \textbf{without the goal}

$$h(G) \overset{\text{def}}{=} \min i \leq n \text{ such that } T \not\models \neg G_i$$

i.e., $h(G)$ encodes first time $i$ at which goal $G$ consistent with $T$

Such $h$ is well defined

- Good news: $h$ very \textbf{informative}; indeed $h(G) = h^*(G)$ (optimal)
- Bad news: $h$ \textbf{intractable}
Consider now approximation \( h_\Gamma \) given by sets \( \Gamma_0, \ldots, \Gamma_n \) of deductive consequences of \( T \) at the various time points 0, \ldots, \( n \):

\[
h_\Gamma(G) \overset{\text{def}}{=} \min \ i \leq n \ \text{such that} \ \Gamma_i \not\models \neg G_i
\]

- If sets \( \Gamma_i = \emptyset \), then \( h_\Gamma(G) = 0 \) (non-informative)
- If sets \( \Gamma_i = PI_i(T) \), then \( h_\Gamma(G) = h(G) \) (intractable)
- Always \( 0 \leq h_\Gamma \leq h \)

**Question:** how to define sets \( \Gamma_i \) so that resulting LBs are informative and tractable?

\( (PI_i(T) = \text{prime implicates of } T \text{ at time } i) \)
Prime Implicates and Lower Bounds: First attempt

Stratify Strips theory $T$ (without the goal) as

$$T = T_0 \cup T_1 \cup \cdots \cup T_m$$

Define sequence of sets $\Gamma_i$ iteratively as

$$\Gamma_0 \overset{\text{def}}{=} PI_0(T_0)$$

$$\Gamma_{i+1} \overset{\text{def}}{=} PI_{i+1}(\Gamma_i \cup T_{i+1})$$

It follows that no info lost in iteration, and same sets and $h$ result:

$$\Gamma_i = PI_i(T)$$

$$h_\Gamma = h = h^*$$

But then computation of $h_\Gamma$ remains intractable . . .
Prime Implicates and Tractable Lower Bounds

Define sequence of sets $\Gamma_i$ iteratively as

$$\Gamma_0 \overset{\text{def}}{=} PI_0^k(T_0)$$
$$\Gamma_{i+1} \overset{\text{def}}{=} PI_{i+1}^k(\Gamma_i \cup T_{i+1})$$

for a fixed $k = 1, 2, \ldots$, where $PI_i^k(T)$ stands for set of prime implicates of $T$ at time $i$ with size no greater than $k$

**Key result:** We show in paper that for Strips theories $T$

- sequence of $\Gamma_i$ sets and $h_\Gamma$ informative and tractable
- $h_\Gamma$ equal to Graphplan $h_G$ for $k = 2$, and
- $x \in \text{Layer}_i$ iff $\neg x_i \notin \Gamma_i$ AND $(x, y) \in \text{Layer}_i$ iff $\neg x_i \lor \neg y_i \in \Gamma_i$

where $x \in \text{Layer}_i$ and $(x, y) \in \text{Layer}_i$ stand for atom and mutex pair in layer $i$ of planning graph
General Framework: Stratified Theories

Propositional theories $T$ defined over indexed variables $x_i \in L_i$, $0 \leq i \leq m$, that can be expressed as union of subtheories $T_0, \ldots, T_m$ where

- $T_0$ made up of clauses $C_0 \in L_0$
- $T_{i+1}$ made up of clauses $C_i \lor C_{i+1}$, where $C_{i+1} \in L_{i+1}$ and $C_i \in L_i$ ($C_{i+1}$ non-empty)

Example: Stratified theory for Strips with horizon $n$

1. **Init** $T_0$: $p_0$ for $p \in I$, and $\neg q_0$ for $q \in A$ not in $I$
2. **Action Layers** $T_{i+1}$: for $i = 0, 2, \ldots, n - 2$
   - $p_i \lor \neg a_{i+1}$ for each $a \in O$ and $p \in \text{pre}(a)$
   - $\neg a_{i+1} \lor \neg a'_{i+1}$ for interfering $a$, $a'$ in $O$
3. **Propositional Layers** $T_{i+1}$: for $i = 1, 3, \ldots, n - 1$
   - $\neg a_i \lor p_{i+1}$ for each $a \in O$ and $p \in \text{add}(a)$
   - $\neg a_i \lor \neg p_{i+1}$ for each $a \in O$ and $p \in \text{del}(a)$
   - $a^1_i \lor a^2_i \lor \cdots \lor a^{np}_i \lor \neg p_{i+1}$ for each $p \in A$
Tractable PI-k Inference over Stratified Theories

Three conditions guarantee that the iterative computation of prime implicates of bounded size remains tractable for stratified theories $T$:

\begin{align*}
\Gamma_0 & \overset{\text{def}}{=} PI_0^k(T_0) \\
\Gamma_{i+1} & \overset{\text{def}}{=} PI_{i+1}^k(\Gamma_i \cup T_{i+1})
\end{align*}

1. $T$ is compiled: resolvents over variables $x_{i+1}$ in $T_{i+1}$ subsumed in $T$

2. $T$ has bounded support width: number of clauses $C_i \lor C_{i+1}$ in $T_{i+1}$ with common literal $l_{i+1} \in C_{i+1}$ and body $|C_i| > 1$, bounded

3. $T$ is pure: only $x_{i+1}$ or $\neg x_{i+1}$ occur in $T_{i+1}$

- Stratified Strips theories are compiled, have support width 1, and can easily be made pure (3. not needed for $k \leq 2$)
- Paper contains sound algorithm for computing $\Gamma_i$ sets that under conditions 1--3 is complete and polynomial
Graphplan vs. Variable Elimination and Variations

- **Variable Elimination** is a family of algorithms for solving SAT, CSPs, Bayesian Networks, etc (Dechter et al) that follows the pattern of **gaussian elimination** for solving linear equations.

- Given a theory $T = T_0$ over variables $x_0, \ldots, x_n$
  
  - **Forward pass:** eliminate var $x_i$ from $T_i$ resulting in theory $T_{i+1}$ over $x_{i+1}, \ldots, x_n$, $0 \leq i < n$
  
  - **Backward pass:** Solve theories $T_n, T_{n-1}, \ldots, T_0$ in order, each for a single variable; result is a model (if $T$ is satisfiable)

-- **Good:** backward pass (solution extraction) is **backtrack free**

-- **Bad:** forward pass (elimination pass) is **exponential** in time and space
Alternative 1: Bounded-k Variable Elimination

• Restricts size of constraints induced by elimination of vars to $k$
• Elimination sound but not complete; performs in polynomial time (removes some but not all backtracks)

Alternative 2: Bounded-k Block Elimination

• Eliminates blocks of vars in one-shot, inducing constraints of size $\leq k$ only
• Stronger than Bounded-k Var Elimination, but exponential in size of blocks
Graphplan and Bounded-k Elimination

As a corollary of earlier results we get that:

- For Strips theories, Bounded-k Block Elimination is polynomial in the size of the blocks (blocks are the sets of vars in same layer)
- Graphplan actually does a Bounded-2 Block Elimination pass forward exactly, followed by a backward Backtrack Search
- Thus Graphplan fits nicely in the variable elimination framework, where it exploits the special structure of Strips theories
Negative vs. Positive Deductive Lower Bound

LB scheme based on proving negation of the goal

\[ h(G) \overset{\text{def}}{=} \min i \leq n \text{ such that } T \not\models \neg G_i \]

\( h(G) \) is a LB because

if \( \exists \) Plan that achieves \( G \) in \( m \leq n \) steps,
then \( \exists M \) of \( T \land G_m \),
then \( T \not\models \neg G_m \)

**Question:** Can we define LBs based on the proving the goal itself, possibly from transformed theory \( T^+ \)?

\[ h^+(G) \overset{\text{def}}{=} \min i \leq n \text{ such that } T^+ \models G_i \]
A Positive Deductive Lower Bound

• Define $T^+$ as Strips encoding (without the goal) but with
  – deletes removed
  – all possible actions applied: $prec(a)_i \supset a_i$

• Then it turns out
  – $T^+$ consistent and tractable
  – $T^+ \models G_i$ iff $\Gamma_i \not\models G_i$ for $k = 1$

• Thus
  – Positive and Negative LBs coincide for $k = 1$
  – Positive LBs weaker than negative ones for $k > 1$
  – Nonetheless former useful in non-Strips settings . . .
Positive Deductive LBs when Information is Incomplete

\[ h^+(G) \stackrel{\text{def}}{=} \min i \leq n \text{ such that } T^+ \models G_i \]

- With incomplete info, test \( T^+ \models G_i \) intractable

- Still heuristic \( h^{++} \) defined as

\[ h^{++}(G) \stackrel{\text{def}}{=} \min i \leq n \text{ such that } T^{++} \models G_i \]

for any theory \( T^{++} \) stronger than \( T^+ \) remains a LB

- Thus tractable LB can be obtained by mapping \( T^+ \) into stronger and tractable \( T^{++} \)

- So ‘bounds’ in Planning and Knowledge Compilation (Kautz and Selman) related after all . . .

- Indeed, \( h \) used in Brafman-Hoffmann ICAPS 04, can be understood in terms of a compilation of \( T^+ \) into a 2-CNF theory \( T^{++} \) (which is not necessarily the 2-CNF LUB of \( T^+ \))
Summary

- Framework: Iterative computation of prime-implicates of bounded size over stratified theories

- Conditions under which this computation is tractable; Strips theories as special case

- Correspondence with planning graph computation and weak forms of variable elimination

- Positive vs. Negative Deductive Lower bounds

- Uses beyond Strips: conditional effects; incomplete information