Closed-form analysis of dual-branch switched diversity with binary nonorthogonal signalling

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An exact closed-form analytical expression is derived for the average bit error probability of dual-branch switched diversity over Rayleigh fading. A practical scheme is considered employing binary nonorthogonal signalling with noncoherent detection. The analytical results are useful to evaluate the performance–bandwidth trade-off in systems that intentionally employ nonorthogonal signalling.

Introduction: Switched diversity has been thoroughly studied as an attempt at simplifying practical systems exploiting diversity. The simplest and best studied switched diversity system is the dual-branch switch-and- stay combining (SSC) over independent and identically distributed (i.i.d.) branches [1, Chap. 9]. Noncoherent detection of binary signals is frequently adopted in practical SSC systems as one of the least complex modulation schemes. In such a case, signals can be chosen nonorthogonal at the transmitter in order to reduce bandwidth utilisation, at the expense of certain performance degradation [2, Chap. 5].

Considerable attention has been paid to the performance analysis of SSC over i.i.d. fading channels [1, 3–5]. By following the moment-generating function (MGF) approach, results in [1] and [3] for the average bit error probability (BEP) in the form of single finite integrals. In [4], a new analysis is performed to give a closed-form BEP expression for coherent detection. An exact formula for noncoherent detection with orthogonal signalling is provided in [5]. However, to the best of the authors’ knowledge, exact closed-form expressions for noncoherent detection of correlated signals are not found in the literature.

In this Letter, a new closed-form BEP analysis is presented for noncoherent detection of correlated binary signals with dual-branch switched diversity over i.i.d. Rayleigh fading. The resultant average BEP expression is in the form of Marcum Q and elementary functions, thus avoiding the need for numerical integration.

Average bit error probability: Let us assume a dual-branch SSC system under i.i.d. Rayleigh fading. The probability density function (pdf) of the instantaneous signal-to-noise ratio (SNR) per symbol $\gamma_S$ at the output of the combiner is [1, eqn. (9.275)]

$$f_{\gamma_S}(x) = \begin{cases} \frac{1}{\gamma} \left[ 1 - \exp\left( -\frac{\gamma_S x}{\gamma} \right) \right] \exp\left( -\frac{x}{\gamma} \right), & x < \gamma_T \\ \frac{1}{\gamma} \left[ 1 - \exp\left( -\frac{\gamma_S x}{\gamma} \right) \right] \exp\left( -\frac{x}{\gamma} \right), & x \geq \gamma_T \end{cases}$$

where $\gamma_T$ is the average SNR on each diversity branch and $\gamma_T$ is the switching threshold. Given the conditional BEP $P_b(x) = \Pr[\text{bit error} | \gamma_S = x]$ the average BEP is calculated by

$$\tilde{P}_b = \int_0^{\infty} P_b(x) f_{\gamma_S}(x) dx$$

After considering [6, eqns. (40) and (42)], the conditional BEP for noncoherent binary signalling given in [1, eqn. (8.70)] can be expressed in terms of the symmetric difference of Marcum Q functions, i.e.

$$P_b(x) = \frac{1}{2} \left[ 1 - Q(b \sqrt{x}, a \sqrt{x}) + Q(a \sqrt{x}, b \sqrt{x}) \right]$$

with $a = (1 - \sqrt{1 - \rho^2})/2$ and $b = (1 + \sqrt{1 - \rho^2})/2 = \rho/(2a)$, where $\rho$ is the magnitude of the cross-correlation between the two signals.

Substituting (1) and (3) into (2) and after some algebraic manipulations the following expression for the average BEP is obtained:

$$\tilde{P}_b = \frac{1}{2} \left[ 2 - 2 e^{-\gamma_T} \right] [\gamma - U(b, a, \gamma) + U(a, b, \gamma)]$$

where $U(a, b, \gamma)$ is given by

$$U(a, b, \gamma) = \int_a^\gamma e^{-\gamma} Q(a \sqrt{\gamma}, b \sqrt{\gamma}) d\gamma$$

and $V(\gamma; a, b, \gamma)$ is defined as

$$V(\gamma; a, b, \gamma) = \int_0^\gamma e^{-\gamma} Q(a \sqrt{\gamma}, b \sqrt{\gamma}) d\gamma$$

What remains to complete the analysis is to show that $U$ and $V$ may be given in exact closed-form by a finite combination of Marcum Q and elementary functions.

The complete integral $U$ is crucial for performance analysis of noncoherent modulations in fading channels. In [7], an exact closed-form expression for $U$ was derived in terms of Gauss hypergeometric functions, which can be easily expressed in terms of Legendre polynomials. Then, after some algebra the following symmetric difference of $U$ functions is obtained:

$$U(a, b, \gamma) - V(b, a, \gamma) = \frac{\gamma}{\sqrt{\rho^2 - \gamma^2}} (a^2 - b^2)$$

where $\gamma = 1 + 1/\sqrt{\rho}$.

On the other hand, the integral $V$ can be studied within the theory of the incomplete cylindrical functions developed by Agrest and Maksimov [8]. Generic forms of the incomplete integral $V$ have been recently investigated in [9]. After simple rescaling and further simplifications, lemma 1 of [9] can be exploited to obtain the following symmetric difference of $V$ functions:

$$V(a, b, \gamma) - V(b, a, \gamma) = \frac{\gamma}{\sqrt{\rho^2 - \gamma^2}} (a^2 - b^2)$$

with $a = \rho/(2\sqrt{c + \sqrt{c^2 - \rho^2}})$. Then, after substituting (8) into (7), the two symmetric differences in (6) and (7) can be replaced in (4) to obtain the final average BEP expression:

$$\tilde{P}_b(\gamma; \rho, \gamma_T) = \frac{1}{2} \left[ 1 + e^{-\gamma_T/2} \left( f(\gamma_T) + e^{-\gamma_T} \right) \right]$$

where $f(\gamma_T) = \rho/y_T$ and $y_T = \sqrt{1 - \rho^2}$. Note that the previous expression is given in closed-form in terms of Marcum Q and elementary functions. Given that analytical properties of Marcum Q function are well-studied, obtaining further insight from (9) is straightforward, e.g., simple upper bounds or asymptotic approximations [1, Sect. 4.2]. Moreover, the presented analytical approach is directly applicable to any modulation with conditional BEP in the form given in (3) (e.g., differentially coherent modulation such as DQPSK).
Numerical results: The average BEP expression in (9) has been evaluated in order to show the performance–bandwidth trade-off for correlated binary frequency shift keying (FSK) signals. The switching threshold that minimises the average BEP has been obtained by means of standard numerical minimisation techniques. Fig. 1 shows the optimum switching threshold against SNR for different values of the cross-correlation. By using the optimum switching threshold, average BEP against average SNR is plotted in Fig. 2 for different values of the cross-correlation parameter. In the same Figure, simulation results are superimposed on the analytical curves.

Conclusions: A simple and elegant closed-form expression for the average BEP of noncoherent and nonorthogonal binary signalling with switched diversity is derived in terms of the symmetric difference of Marcum $Q$ functions. This expression leads to easily computable results, which are useful for the design of switched diversity based systems.

Acknowledgments: This work was partially supported by the Spanish Government and the European Union under project TEC2007-67289/TCM and by the company AT4Wireless S.A.

References