Abstract—Two models derived from the coherent Gaussian channel are studied. In each, information is sent by energy modulation. For continuous energy the model is the additive exponential noise (AEN) channel, whose capacity equals that of a Gaussian channel at identical noise and signal energy levels. The second model is a quantized version of the AEN channel. An upper bound to its capacity is derived and, when photons are used as energy unit, shown to be strictly smaller than for the additive exponential/Gaussian noise channel.

I. THE ADDITIVE ENERGY CHANNEL

We consider several variations on the discrete-time Gaussian channel. In all cases, the output variable at time \( k \), \( z_k \), is

\[ z_k = x_k(s_k) + w_k, \tag{1} \]

made up of a signal component \( x_k \), a function (possibly stochastic) of the corresponding input \( s_k \), and a noise \( w_k \). The input is used under an energy constraint, \( E_s \).

In the coherent AWGN channel both input and output are complex numbers. The variations we study are non-coherent in the sense that information is sent by modulating a signal energy \( |s_k|^2 \) rather than \( s_k \) itself. One motivation is to study how sensitive information-theoretic results are to changes in the underlying model. For example, phase noise or, in general, the cost incurred by carrier synchronization are somewhat difficult to include in an AWGN model.

The AEN channel The additive exponential noise (AEN) channel was studied by Verdú [1]. We establish a correspondence with the AWGN channel by setting the input to \( s_k \) with variance \( \sigma_k^2 \). Here, as in AWGN, the signal output \( x_k \) coincides with the input \( s_k \). The AEN noise \( w_k \) is obtained from the AWGN value by taking the squared modulus and has thus an exponential density of mean \( E_w = \sigma^2 \), where \( \sigma^2 \) is the variance of the Gaussian noise. It is important to remark that \( s_k \triangleq \sigma_k^2 \).

The AE-Q channel The quantized additive energy (AE-Q) channel arises from assuming the existence of a quantum of energy, a primary example being the photon. The variables \( z_k, x_k \) and \( w_k \) are hence non-negative integers. The output \( x_k \) has a Poisson distribution with parameter \( s_k \), which implies that noise is (partly) signal-dependent; the noise \( w_k \) follows a geometric distribution of mean \( \epsilon_w = \sigma^2 = E_w \). The (discrete) energy constraint is denoted by \( \epsilon_s \).

II. BOUNDS TO THE SINGLE-USER CAPACITY

The capacity \( C \) (in nats per channel use) of the AWGN channel and of the equivalent AEN channel [1] is

\[ C = \log(1 + \text{SNR}), \tag{2} \]

where SNR is the signal-to-noise ratio, \( \text{SNR} = E_s/\sigma^2 \) for AWGN and \( \text{SNR} = E_s/E_w \) for the AEN channel.

For the AE-Q channel, we resort to providing tight upper and lower bounds to its capacity \( C(\epsilon_s, \epsilon_w) \). The capacity is upper bounded by

\[ C^+(\epsilon_s, \epsilon_w) = \min(C_L, C_H), \tag{3} \]

where \( C_L \) and \( C_H \) are respectively given by

\[ C_L = H_{\text{Geom}}(\epsilon_s + \epsilon_w) - H_{\text{Geom}}(\epsilon_w) \tag{4} \]

\[ C_H = \log\left(1 + \frac{\sqrt{2} - 1}{\sqrt{1 + 2\epsilon_s}}\right) \tag{5} \]

Here \( H_{\text{Geom}}(t) \) is the entropy of a geometric distribution with mean \( t \), \( H_{\text{Geom}}(t) = (1 + t) \log(1 + t) - t \log t \).

Note that \( C_H \) does not depend on \( \epsilon_w \), as it is due to the Poisson noise. In addition, \( C_H \) behaves as \( 1/2 \log \epsilon_s \) for \( \epsilon_s \to \infty \).

For finite values of \( \epsilon_s \) and \( \epsilon_w \), the capacity of the AE-Q channel is strictly upper bounded by the capacity of an AEN channel with SNR \( \epsilon_s/\epsilon_w \), that is

\[ C(\epsilon_s, \epsilon_w) < \log\left(1 + \frac{\epsilon_s}{\epsilon_w}\right). \tag{6} \]

However, as \( \epsilon_w \to \infty \), the upper bound essentially coincides with the capacity of the additive exponential noise channel,

\[ C_L = \log\left(1 + \frac{\epsilon_s}{\epsilon_w}\right) + O(\epsilon_w^{-1}). \tag{7} \]

Lower bounds are reasonably close to these upper bounds. It is instructive to set the unit energy to \( h\nu \), as for photons at frequency \( \nu \), and set \( \epsilon_w \) to the thermal background noise, the blackbody radiation. In this case, for a fixed frequency \( \nu \), the AE-Q capacity is close to the AWGN below a signal-to-noise ratio threshold; above the capacity-cost function changes from \( \log \epsilon_s^+ \) to \( 1/2 \log \epsilon_s \). For low values of \( \epsilon_s, C_L \) is tight, whereas for higher values of \( \epsilon_s, C_H \) gives the best bound. The transition approximately occurs when \( C_L = C_H \). For large enough photon counts this is almost equivalent to \( \epsilon_s = \epsilon_w^2 \).

REFERENCES