In a related model, where the mobile network also has n static nodes along with n mobile nodes, the optimal tradeoff can be obtained for sufficiently low throughputs. We can show that for any throughput $T(n) = \Theta(1/n^{1/2+\epsilon})$, $\epsilon > 0$, the tradeoff given by $T(n) = \Theta(D(n)/n)$ can be achieved. This is the same as the tradeoff for the fluid model in [2]. This establishes the optimal tradeoff for this range of low throughputs. The scheme achieving this tradeoff uses the scheduling scheme given in this correspondence along with a randomization technique and chasing in a manner similar to Scheme 3(a) in [2]. However, the optimal tradeoff for the mobile network with no static nodes is unknown.

REFERENCES

[1] A. El Gamal, J. Mammen, B. Prabhakar, and D. Shah, "Throughput-delay tradeoff in wireless networks," in *Proc. IEEE INFOCOM*, Hong Kong, Mar. 2004.

[2] ——, "Optimal throughput-delay scaling in wireless networks—Part I: The fluid model," *IEEE Trans. Inf. Theory*, vol. 52, no. 6, pp. 2568–2592, Jun. 2006.

[3] M. Grossglauser and D. M. C. Tse, "Mobility increases the capacity of ad-hoc wireless networks," in *Proc. IEEE INFOCOM*, Anchorage, AK, 2001, pp. 1360–1369.

[4] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Trans. Inf. Theory*, vol. 46, no. 2, pp. 388–404, Mar. 2000.

[5] F. P. Kelly, *Reversibility and Stochastic Networks*. New York: Wiley, 1979.

[6] S. R. Kulkarni and P. Viswanath, "A deterministic approach to throughput scaling in wireless networks," *IEEE Trans. Inf. Theory*, vol. 50, no. 6, pp. 1041–1049, Jun. 2004.

[7] R. Motwani and P. Raghavan, *Randomized Algorithms*. Cambridge, U.K.: Cambridge Univ. Press, 1995.

[8] M. J. Neely and E. Modiano, "Capacity and delay tradeoffs for *ad hoc* mobile networks," *IEEE Trans. Inf. Theory*, vol. 51, no. 6, pp. 1917–1937, Jun. 2005.

[9] R. W. Wolff, *Stochastic Modeling and the Theory of Queues*. Englewood Cliffs, NJ: Prentice-Hall, 1988.

Coding in the Block-Erasure Channel

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***Abstract*—In this correspondence, we study an M-ary block-erasure channel with B blocks, where with probability ϵ a block of L coded symbols is erased. The behavior of the error probability of coded systems over such channels is studied, and we show that, if the code is diversity-wise maximum-distance separable, its word error probability is equal to the outage probability, which admits a very simple expression. This correspondence is intended to complement the error probability analysis in previous work by Lapidoth and shed some light on the design of coding schemes for nonergodic channels.**

***Index Terms*—Diversity, erasure channels, error probability, maximum-distance separable (MDS) codes, maximum-likelihood (ML) decoding, nonergodic channels, outage probability.**

I. INTRODUCTION

The block-erasure channel is a very simplified model of a fading channel where parts of the codeword are completely erased by a deep fade of the channel [1]. This channel corresponds to the large signal-to-noise ratio (SNR) regime of the block-fading channel [2]–[7], and its interest lies on its simplicity and nonergodicity, typical of many real wireless communication systems, such as orthogonal frequency division multiplexing (OFDM) or frequency-hopped systems. Coding for the block-erasure channel with convolutional codes has been studied in some detail in [1]. In this context, non-ergodicity means that the transmitted codeword spans only a finite number B of independent realizations (degrees of freedom) of the channel irrespectively of its length.

In this correspondence we study the problem of *fixed-rate* transmission over the block-erasure channel. This correspondence complements previous error probability analysis for convolutional codes in the block-erasure channel done by Lapidoth in [1]. In particular, we derive simple expressions for the word and bit error probabilities of general codes of a fixed rate, as well as tight bounds. We find that diversity-wise maximum-distance separable (MDS) codes have the lowest possible error probability and are therefore optimal for this channel.

II. CHANNEL MODEL

We study a block-erasure channel with B blocks. With probability ϵ a block of L symbols is completely erased and with probability $1 - \epsilon$ a block of L coded symbols is received correctly (noiseless sub-channel), independently from block to block. Consider the transmission of an M-ary code $\mathcal{C}$ of length $N = BL$ and rate $R = \frac{K}{N}$ bits per channel use, where $K = \log_2 |\mathcal{C}|$. Also, let $\mathbf{x} = (\mathbf{x}_1, \dots, \mathbf{x}_B) \in \{0, 1, \dots, M-1\}^N$ be a codeword of $\mathcal{C}$. We denote erasures by "?." The block-erasure channel is illustrated in Fig. 1.

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Color version of Fig. 3 is available online at http://ieeexplore.ieee.org.
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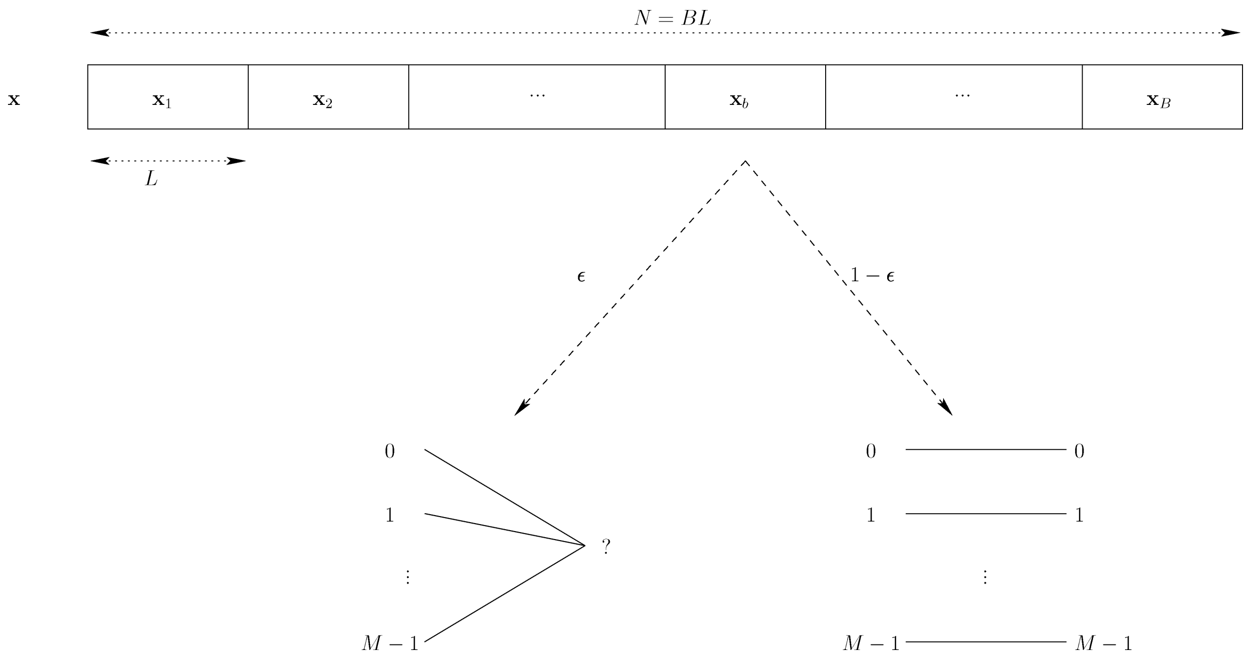

Fig. 1. The block-erasure channel with B blocks. The symbols of block b, $b = 1, \ldots, B$ are erased with probability ϵ. The symbols of block b, $b = 1, \ldots, B$ are received correctly (noiseless channel) with probability $1 - \epsilon$.

Define the erasure pattern vector $\mathbf{e} = (e_1, e_2, \ldots, e_B) \in \{0,1\}^B$, whose b-th component is $e_b = 1$ if the block is erased and $e_b = 0$ otherwise. Thus, $P(e_b = 1) = \epsilon$ and $P(e_b = 0) = 1 - \epsilon$, namely, the components of the erasure pattern $\mathbf{e}$ are i.i.d. Bernoulli random variables (with success probability ϵ). We assume that the receiver has channel-state information (CSI), i.e., the receiver knows the erasure pattern $\mathbf{e}$.[1]

III. Error Probability Analysis

In this section we define the *word* and *bit* error probabilities of coded schemes over the block-erasure channel described in the previous section. We also discuss the information theoretic limits of the channel.

We define the word error probability as the probability of decoding in favor of a codeword $\hat{\mathbf{x}}$ when codeword $\mathbf{x}$ was transmitted, averaged over all possible transmitted codewords $\mathbf{x} \in \mathcal{C}$

$$P_e^w(\epsilon) = \frac{1}{|\mathcal{C}|} \sum_{\hat{\mathbf{x}} \neq \mathbf{x}} \Pr\{\hat{\mathbf{x}} \neq \mathbf{x}\}. \tag{1}$$

We further consider linear codes only, and thus, the error probability does not depend on the transmitted codeword. We then assume the transmission of the all-zero codeword, i.e., $\mathbf{x} = (0, \ldots, 0)$. Consider the *maximum-likelihood* (ML) decoder

$$\hat{\mathbf{x}} = \arg\max_{\mathbf{x} \in \mathcal{C}} p(\mathbf{y}|\mathbf{x}) \tag{2}$$

and define the subsets

$$\mathcal{C}(\mathbf{e}) = \{\mathbf{x} \in \mathcal{C} \ | \ \text{if} \ \ e_b = 0, \ \ \mathbf{x}_b = (0, \ldots, 0) \ \ \forall b \in (1, \ldots, B)\} \tag{3}$$

as the set of codewords that have nonzero components in the erased positions *only*. Obviously, the transmitted codeword belongs to $\mathcal{C}(\mathbf{e})$ and by definition $|\mathcal{C}(\mathbf{e})| \geq 1, \forall \mathbf{e} \in \mathbb{F}_2^B$. In words, $\mathcal{C}(\mathbf{e})$ is the set of codewords that, once erased by a given erasure pattern, look identical to the receiver. In such a case, the ML decoder will resolve the ties evenly, and will make an error with probability [1]

$$P_e^w(\mathbf{e}) = 1 - \frac{1}{|\mathcal{C}(\mathbf{e})|} \tag{4}$$

which implies that

$$P_e^w(\epsilon) = \mathbb{E}[P_e^w(\mathbf{e})] = \mathbb{E}\left[1 - \frac{1}{|\mathcal{C}(\mathbf{e})|}\right]. \tag{5}$$

We remark that the only source of error (randomness) in the decoding process is essentially how the ML decoder resolves the ties between the equaly likely candidates in $\mathcal{C}(\mathbf{e})$.

We further define the average bit error probability as

$$P_e^b(\epsilon) = \frac{1}{K} \sum_{k=1}^{K} P_{e,k}(\epsilon) \tag{6}$$

where $P_{e,k}(\epsilon)$ is the probability of error of the k-th information bit.

Definition 1: The block-diversity of a code is defined as

$$\delta = \min_{\substack{\mathbf{x} \in \mathcal{C} \\ \mathbf{x} \neq \mathbf{0}}} |\{b \in (1, \ldots, B) \ | \ \mathbf{x}_b \neq \mathbf{0}\}|. \tag{7}$$

In words, δ represents the limit number of erased blocks that $\mathcal{C}$ can tolerate. Specifically, for a fixed erasure pattern $\mathbf{e}$, if $\delta \leq \sum_{b=1}^{B} e_b$, then $|\mathcal{C}(\mathbf{e})| > 1$ and the ML decoder will make an error with probability $1 - \frac{1}{|\mathcal{C}(\mathbf{e})|}$. Obviously, $\delta \leq B$. If $\delta = B$ we say that $\mathcal{C}$ has *full diversity*. The definition of δ shows that it corresponds to the minimum distance of a code of length B constructed over an alphabet of size M^L. Therefore, by using the Singleton bound [14], we get [4]

$$\delta \leq \delta_B \tag{8}$$

where

$$\delta_B \triangleq 1 + \left\lfloor B\left(1 - \frac{R}{\log_2 M}\right)\right\rfloor. \tag{9}$$

[1]Strictly speaking, this assumption is implicit in the channel model, since the output alphabet includes the erasure symbol.

The Singleton bound states that given B, R and M, the block diversity cannot be larger than (9), and thus full diversity is only guaranteed if $\frac{R}{\log_2 M} \leq \frac{1}{B}$.

Definition 2: A code $\mathcal{C}$ is blockwise MDS if it meets the Singleton bound with equality, i.e., $\delta = \delta_B$.

In the following, we elaborate on the optimality of blockwise MDS codes over the block-erasure channel.

A. Outage Probability

Similarly to other nonergodic channels, the block-erasure channel has zero capacity in the strict Shannon sense, since with probability ϵ^B all channels are erased and reliable communication is not possible. We write the mutual information between the input and output of the channel for a given erasure pattern $\mathbf{e}$ as

$$I(\mathbf{e}) = \frac{1}{B}\sum_{b=1}^{B} \bar{e}_b \log_2 M \qquad \text{(bits per channel use)} \tag{10}$$

where $\bar{e}_b$ denotes the binary complement of e_b. This comes from the fact that the block-erasure channel is nothing but a set of B parallel channels (used an equal fraction of the time), each conveying either $\log_2 M$ bits per channel use if $e_b = 0$ or none if $e_b = 1$. Note that, since B is *finite*, $I(\mathbf{e})$ is a random variable. If $B \to \infty$ the channel the distribution of $I(\mathbf{e})$ becomes a function with a step at the channel capacity $\log_2 M(1-\epsilon)$, and the channel becomes an *ergodic* M-ary erasure channel.

We define the information outage probability as the probability that the transmission rate R is not supported by a given channel realization

$$P_{\rm out}(\epsilon) \triangleq \Pr\{I(\mathbf{e}) < R\}. \tag{11}$$

In such nonergodic channels, $P_{\rm out}(\epsilon)$ is then the *best possible* word error probability.[2]

We have the following result.

Proposition 1: Consider the transmission M-ary codes over the block-erasure channel. Then

$$\lim_{\epsilon\to 0} P_{\rm out}(\epsilon) = \binom{B}{\left\lceil \frac{BR}{\log_2 M}\right\rceil - 1}\epsilon^{\delta_B}. \tag{12}$$

Proof: We can write the outage probability as

$$\begin{aligned} P_{\rm out}(\epsilon) &= \Pr\{I(\mathbf{e}) < R\} \\ &= \Pr\left\{\frac{1}{B}\sum_{b=1}^{B}\bar{e}_b \log_2 M < R\right\} \\ &= \Pr\left\{\sum_{b=1}^{B}\bar{e}_b < \frac{BR}{\log_2 M}\right\} \\ &= \Pr\left\{A \leq \left\lceil \frac{BR}{\log_2 M}\right\rceil - 1\right\} \\ &= \sum_{k=0}^{\left\lceil \frac{BR}{\log_2 M}\right\rceil - 1} \binom{B}{k}(1-\epsilon)^k \epsilon^{B-k} \end{aligned} \tag{13}$$

where $A \triangleq \sum_{b=1}^{B} \bar{e}_b$ is a binomial random variable with success probability $1-\epsilon$. We have quite trivially expressed the outage probability as the cumulative distribution function (cdf) of a binomial random variable. Therefore, we clearly get that

$$\begin{aligned} \lim_{\epsilon\to 0} P_{\rm out}(\epsilon) &= \binom{B}{\left\lceil \frac{BR}{\log_2 M}\right\rceil - 1}\epsilon^{B-\left\lceil \frac{BR}{\log_2 M}\right\rceil+1} \\ &= \binom{B}{\left\lceil \frac{BR}{\log_2 M}\right\rceil - 1}\epsilon^{B+\left\lfloor -\frac{BR}{\log_2 M}\right\rfloor+1} \\ &= \binom{B}{\left\lceil \frac{BR}{\log_2 M}\right\rceil - 1}\epsilon^{1+\left\lfloor B\left(1-\frac{R}{\log_2 M}\right)\right\rfloor} \end{aligned} \tag{14}$$

which shows the result. □

Remark 1: The outage probability has slope δ_B for low ϵ in a log-log scale, and asymptotic coding gain $\binom{B}{\lceil \frac{BR}{\log_2 M}\rceil - 1}$. Thus it clearly corresponds to the high SNR regime of a block-fading channel [7].

Remark 2: If $\frac{R}{\log_2 M} = \frac{1}{B}$ (full diversity), $P_{\rm out}(\epsilon) = \epsilon^B$, i.e., the probability that the rate R is not supported by the channel is equal to the probability of having all the blocks erased.

Fig. 2 shows the outage probability and the asymptotic limit (12) for $R = \frac{1}{2}$ binary codes ($M = 2$) over a block erasure channel with $B = 2, 4$ and 8 blocks.

B. Word Errors

The previous result proves the optimality (in diversity only) of designing MDS codes for such channels. In order to achieve the optimal performance, we start from

$$P_e^w(\mathbf{e}) = 1 - \frac{1}{|\mathcal{C}(\mathbf{e})|}. \tag{15}$$

For any code (with a given block diversity $\delta \leq \delta_B$), since $|\mathcal{C}(\mathbf{e})| \leq |\mathcal{C}|$ we can trivially upper-bound (15) (for large block length) as

$$P_e^w(\mathbf{e}) \leq \begin{cases} 1, & \text{if } \sum_{b=1}^{B} e_b \geq \delta \\ 0, & \text{if } \sum_{b=1}^{B} e_b < \delta \end{cases} \tag{16}$$

which leads to

$$\begin{aligned} P_e^w(\epsilon) &\leq \Pr\left\{\sum_{b=1}^{B} e_b \geq \delta\right\} \\ &= \Pr\left\{\sum_{b=1}^{B} \bar{e}_b \leq B-\delta\right\} \\ &= \sum_{k=0}^{B-\delta}\binom{B}{k}(1-\epsilon)^k\epsilon^{B-k}. \end{aligned} \tag{17}$$

In general (17) is not necessarily tight. However (and possibly surprisingly), if $\mathcal{C}$ is MDS, i.e., $\delta = \delta_B$, the bound is tight since (17) coincides with $P_{\rm out}(\epsilon)$. In other words, if $\mathcal{C}$ is MDS, its word error probability is given by the outage probability, since the decoder will decode correctly under all erasure patterns such that $\sum_{b=1}^{B} e_b < \delta_B$.

Fig. 3 confirms the above discussion. We have plotted the outage probability and the limiting behavior (12), as well as the word error rate (WER) simulations for the $(23,33)_8$ and $(133,171)_8$ convolutional codes with $L = 25$ (circles/crosses) and $L = 2500$ (diamonds/squares) respectively. As we observe, the simulated WER of the different codes for the different block lengths is the same and matches perfectly with the outage probability.

Remark 3: This result can be a priori surprising, since it characterizes the performance of **any** MDS code of any (sufficiently large) block length over the block erasure channel. A posteriori, the result

[2]Remark that this is only true for sufficiently large block length. In general, Fano's inequality gives [8]–[10]

$$P_e^w(\epsilon) \geq \mathbb{E}\left[\left|1 - \frac{I(\mathbf{e})}{R} - \frac{1}{BLR}\right|_+\right]$$

where $|x|_+ = \max\{0, x\}$.

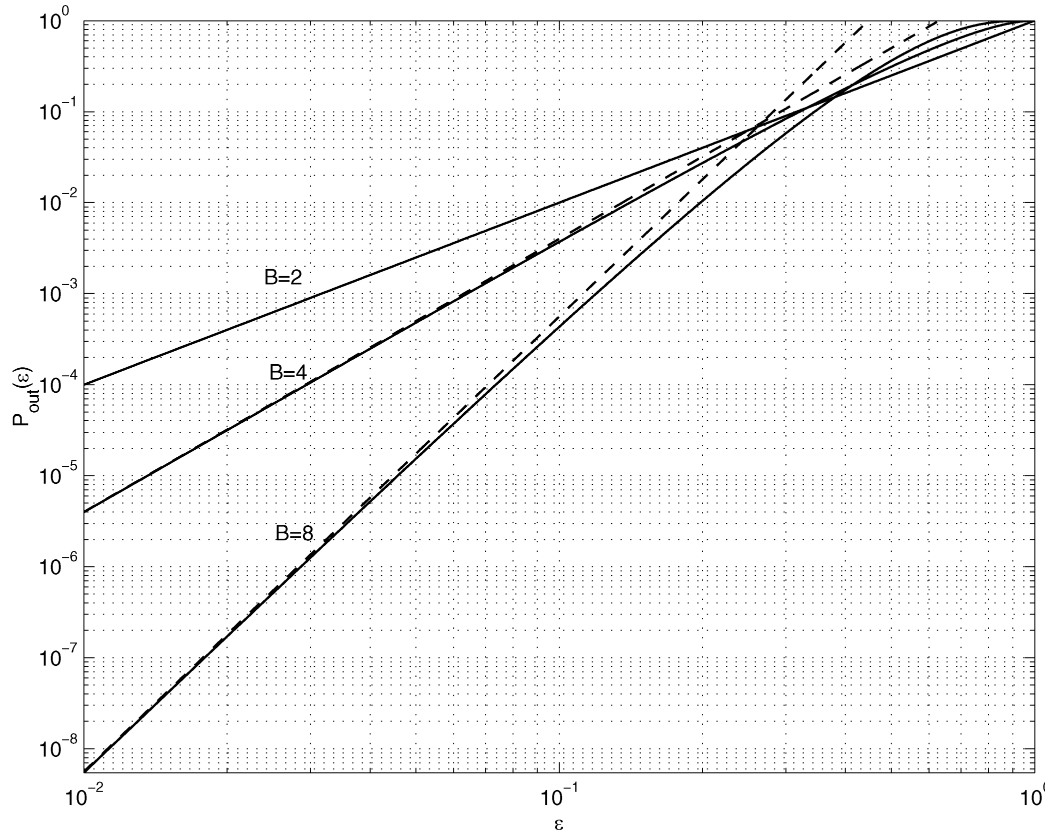

Fig. 2. Outage probability (solid lines) and (12) (dashed lines) in a log-log scale in a block-erasure channel with $B = 2$, $B = 4$ and $B = 8$ blocks for $R = \frac{1}{2}$ and $M = 2$. The Singleton bound gives $\delta_B = 2, 3$ and 5 respectively.

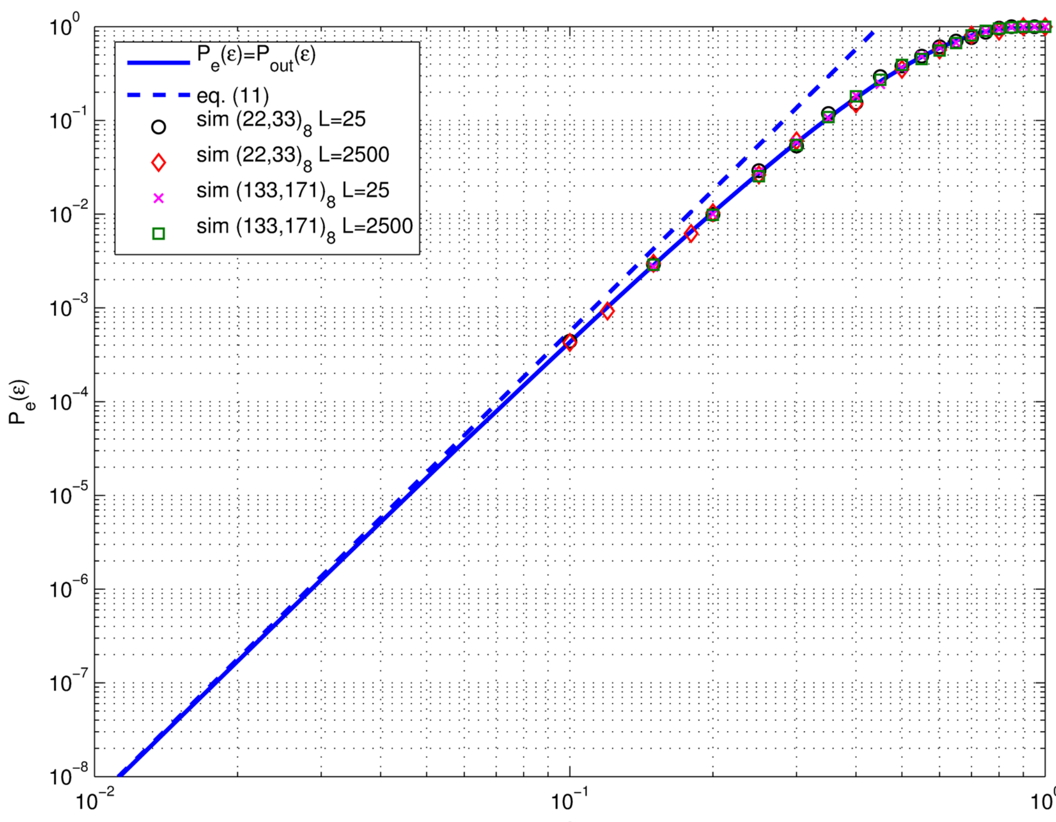

Fig. 3. Outage probability (solid lines), (12) (dashed lines) and simulations with the $(23, 33)_8$ and $(133,171)_8$ convolutional codes with $L = 25$ (corresponds to 100 information bits per codeword) and $L = 2500$ (corresponds to 10000 information bits per codeword) in log-log scale in a block-erasure channel with $B = 8$ blocks.

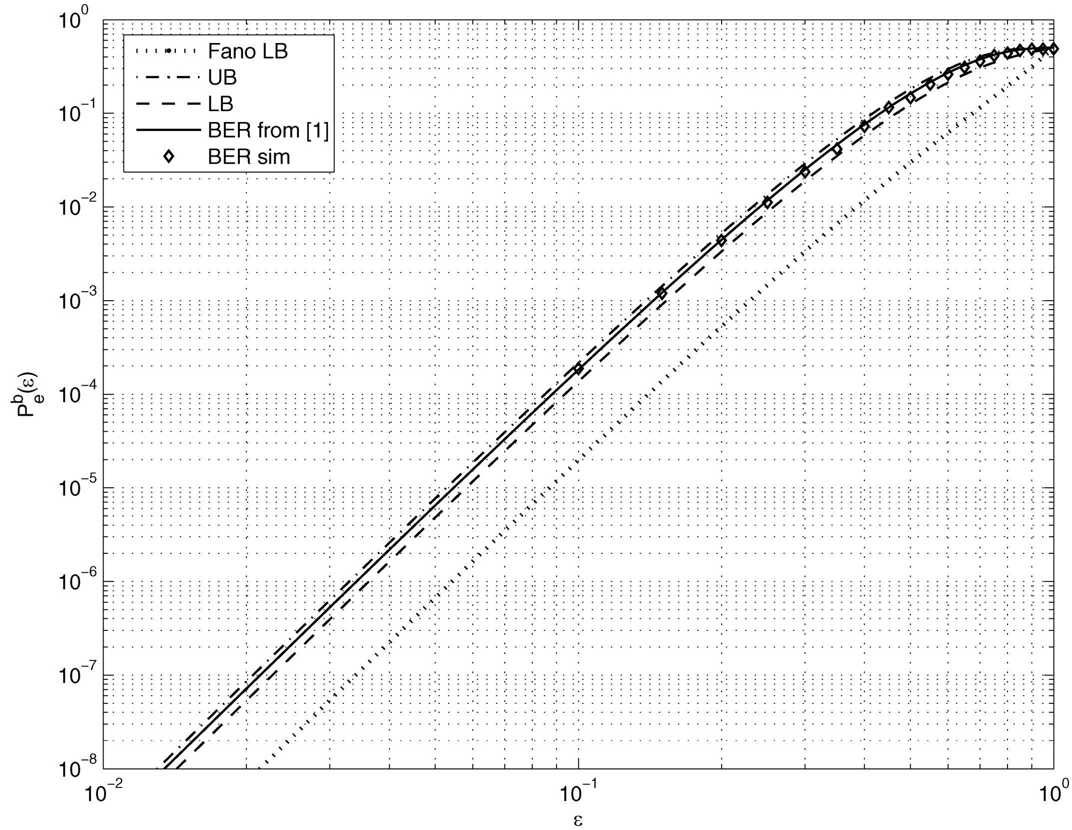

Fig. 4. Bit-error probability in log-log scale in a block-erasure channel with $B = 8$ blocks for $R = 1/2$. We show the Fano lower bound (21) (dotted line), the upper bound (19) (dash-dotted line), the lower bound (24) (dotted line), the analytical BER expression for the $(23, 33)_8$ code from [1] (solid line) and the BER simulation (diamonds).

seems rather obvious, since it clearly follows as an artifact from the channel model and the definition of MDS codes. We should not then be misled by this, since in realistic non-ergodic block-fading noisy channels MDS codes are necessary, but not sufficient to approach the outage probability [7]. For example, the WER of convolutional codes in the block-fading channel increases with the block length, while the WER of concatenated MDS codes (as the blockwise concatenated codes of [7] or the parallel turbo-codes of [11]) is given by the distribution of the decoding threshold [7], [12].

C. Bit Errors

In this section we show that blockwise MDS codes are also optimal for the bit error probability. We start with a very simple upper bound

$$P_e^b(\mathbf{e}) \leq \begin{cases} \frac{1}{2}, & \text{if } \sum_{b=1}^{B} e_b \geq \delta \\ 0, & \text{if } \sum_{b=1}^{B} e_b < \delta \end{cases} \tag{18}$$

which yields that

$$\begin{aligned} P_e^b(\epsilon) &= \mathbb{E}[P_e^b(\mathbf{e})] \\ &\leq \frac{1}{2} \Pr\left\{ \sum_{b=1}^{B} e_b \geq \delta \right\} \\ &= \frac{1}{2} P_e^w(\epsilon). \end{aligned} \tag{19}$$

By using the bit-error version of Fano's inequality [9, Theorem 4.3.2] (using the fact that the encoder's inputs are bits) we can lower-bound the bit-error probability and get that[3]

$$P_e^b(\mathbf{e}) \geq h^{-1}\left(\left| 1 - \frac{I(\mathbf{e})}{R} \right|_+ \right) \tag{20}$$

where $h(p) = p\log_2 \frac{1}{p} + (1-p)\log_2 \frac{1}{1-p}$ is the binary entropy function and $p = h^{-1}(x)$ denotes the probability p for which $h(p) = x$. Therefore, we get that

$$\begin{aligned} P_e^b(\epsilon) &= \mathbb{E}[P_e^b(\mathbf{e})] \\ &\geq \mathbb{E}\left[h^{-1}\left(\left| 1 - \frac{I(\mathbf{e})}{R} \right|_+ \right) \right] \\ &= \mathbb{E}\left[h^{-1}\left(\left| 1 - \frac{\log_2 M \sum_{b=1}^{B} \bar{e}_b}{BR} \right|_+ \right) \right] \\ &= \sum_{k=0}^{\left\lceil \frac{BR}{\log_2 M} \right\rceil - 1} h^{-1}\left(\left| 1 - \frac{\log_2 M}{BR} k \right|_+ \right) \binom{B}{k} (1-\epsilon)^k \epsilon^{B-k} \end{aligned} \tag{21}$$

since

$$h^{-1}\left(\left| 1 - \frac{\log_2 M \sum_{b=1}^{B} \bar{e}_b}{BR} \right|_+ \right) = 0 \text{ when } \sum_{b=1}^{B} \bar{e}_b \geq \frac{BR}{\log_2 M}. \tag{22}$$

[3]This lower bound was also obtained in [13] for the multiple-antenna case.

Therefore, the bit-error probability has the same slope, namely, the Singleton bound δ_B and this slope is again achievable with MDS codes. We can also lower-bound $P_e^b(\epsilon|\mathbf{e})$ as

$$P_e^b(\mathbf{e}) \geq \begin{cases} \frac{1}{2}\frac{B-\sum_{b=1}^{B}\bar{e}_b}{B}, & \text{if } \sum_{b=1}^{B}\bar{e}_b < \frac{BR}{\log_2 M} \\ 0, & \text{otherwise} \end{cases} \quad (23)$$

which yields to

$$\begin{aligned} P_e^b(\epsilon) &= \mathbb{E}[P_e^b(\mathbf{e})] \\ &\geq \sum_{k=0}^{\left\lceil \frac{BR}{\log_2 M}\right\rceil -1} \frac{1}{2}\frac{B-\sum_{b=1}^{B}\bar{e}_b}{B}\binom{B}{k}(1-\epsilon)^k\epsilon^{B-k} \end{aligned} \quad (24)$$

since the best the decoder can do in case of an outage event to guess a fraction $\frac{B-\sum_{b=1}^{B}e_b}{B}$ of the bits and correct all the others. Depending on the structure of the code, the decoder will do worse than that. For example, in the ML decoder of a convolutional code will choose a wrong path through the trellis, which will yield more errors in the bits corresponding to the nonerased blocks.

Remark 4: The maximum exponent of ϵ in the BER expression for the $(23, 33)_8$ code with periodic interleaving in [1, p. 1470]

$$P_e^b(\epsilon) = 23.5\epsilon^5(1-\epsilon)^3 + 13.5\epsilon^6(1-\epsilon)^2 + 4\epsilon^7(1-\epsilon) + 0.5\epsilon^8 \quad (25)$$

should not come as a surprise, since the code with periodic interleaving is MDS, and thus, $\delta_B = 5$ (the results in [1] are plotted in a linear scale for ϵ and the effect of the slope is not evident). It should also be clear that a random interleaver yields a non-MDS convolutional code, and hence its error probability has a worse slope.

Remark 5: The upper bound (19) and the lower bounds (21) and (24) coincide for full diversity codes.

Fig. 4 shows several bounds and simulations of the bit-error probability. As we see, the difference between the two lower bounds is quite remarkable, which indicates that (21) might not be achievable in general.

IV. Conclusion

A rather simple analysis reveals the usefulness of blockwise MDS codes for the nonergodic block-erasure channel. In this correspondence, it is shown that these codes are optimal in this channel. Expressions of the corresponding word error rates as well as tight bounds on the bit error rate are derived.

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References

[1] A. Lapidoth, "The performance of convolutional codes on the block erasure channel using various finite interleaving techniques," *IEEE Trans. Inf. Theory*, vol. 40, no. 5, pp. 1459–1473, Sep. 1994.

[2] L. H. Ozarow, S. Shamai, and A. D. Wyner, "Information theoretic considerations for cellular mobile radio," *IEEE Trans. Veh. Technol.*, vol. 43, no. 2, pp. 359–378, May 1994.

[3] E. Biglieri, J. Proakis, and S. Shamai, "Fading channels: Information-theoretic and communications aspects," *IEEE Trans. Inf. Theory*, vol. 44, no. 6, pp. 2619–2692, Oct. 1998.

[4] R. Knopp and P. Humblet, "On coding for block fading channels," *IEEE Trans. Inf. Theory*, vol. 46, no. 1, pp. 1643–1646, Jul. 1999.

[5] E. Malkamaki and H. Leib, "Evaluating the performance of convolutional codes over block fading channels," *IEEE Trans. Inf. Theory*, vol. 45, no. 5, pp. 1643–1646, Jul. 1999.

[6] S. Wilson, Y. Leung, and J. Ketchum, "Multifrequency trellis coding with low delay for fading channels," *IEEE Trans. Commun.*, vol. 41, no. 10, pp. 1450–1458, Oct. 1993.

[7] A. Guillén i Fàbregas and G. Caire, "Coded modulation in the block-fading channel: Coding theorems and code construction," *IEEE Trans. Inf. Theory*, vol. 52, no. 1, pp. 91–114, Jan. 2006.

[8] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. New York: Wiley Series in Telecommunications, 1991.

[9] R. G. Gallager, *Reliable Communication and Information Theory*. New York: Wiley , 1968.

[10] L. Zheng and D. Tse, "Diversity and multiplexing: A fundamental tradeoff in multiple antenna channels," *IEEE Trans. Inf. Theory*, vol. 49, no. 5, May 2003.

[11] J. Boutros, E. Calvanese Strinati, and A. Guillén i Fàbregas, "Turbo code design for block-fading channels," in *Proc. 42nd Annu. Allerton Conf. Commun., Contr. Comput.*, Allerton, IL, Sep.-Oct. 2004.

[12] J. Boutros, A. Guillén i Fàbregas, and E. Calvanese Strinati, "Analysis of coding on nonergodic fading fading channels," in *Proc. 43rd Annu. Allerton Conf. Commun., Contr. Comput.*, Allerton, IL, Sep. 2005.

[13] G. Caire and H. El Gamal, "A lower bound on BER for MIMO quasistatic fading channels," Jul. 2003, Private Communication.

[14] R. E. Blahut, *Algebraic Codes for Data Transmission*. Cambridge, U.K.: Cambridge Univ. Press, 2003.