Location-Based Resource Allocation for OFDMA Cognitive Radio Systems

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Introduction

Resource Allocation

Performance Analysis

Practical Scenarios
3 models for cognitive radios:

- **Interweave**: use primary channels only of non used.
- **Underlay**: use all primary channels under interference condition.
- **Overlay**: interference cancellation via collaboration between PU and SU.

⇒ **Mixed strategy**:

- if channel non used by PU: SU transmit without limit.
- if channel used by PU: SU transmit under interference constraint.
Utility of Location Information

- Cognitive radio need an accurate estimation of the channel to the primary users.
- In literature, usually we assume the knowledge of Channel State Information (CSI) between Secondary and Primary users. ⇒ Impractical hypothesis in cognitive radio systems
- Without CSI, Location information can be used to estimate the interference.
Interference estimation from LI

- Pathloss and shadowing model
  \[
P_{rx}(d) = \frac{P_{tx} \xi 10^{0.1X}}{d^\eta},
\]

- Probabilistic interference constraint
  \[
  \Pr\left[10 \log P_{rx}(d) > 10 \log l_{th}\right] \leq p_\epsilon,
  \]

- Constraint on the transmitted power
  \[
  \frac{P_{tx} \xi}{d^\eta} \leq \frac{l_{th}}{10Q^{-1}(p_\epsilon \sigma_x)}.
  \]
System model

- OFDMA-based Cognitive Radio System
- $L$ subcarriers shared between primary and secondary networks
- Cellular environment (base station + users) ⇒ 2 different scenarios: Downlink and Uplink
- Single secondary network (1 base station + $K$ users)
- Multiple primary networks ($N$ base stations + $N$ users)
- The primary users subchannel occupancy is known by spectrum sensing
- Location information is known to the cognitive radio networks
  - by cooperative measurements,
  - by geo-location database.
Problem formulation

\[
\max \sum_{k=1}^{K} \sum_{i=1}^{L} a_{k,i} \log_2 \left( 1 + \frac{|h_{k,i}|^2 p_{k,i}}{N_0} \right)
\]

subject to

\[
\sum_{k=1}^{K} a_{k,i} = 1, \quad \forall i,
\]

\[
\sum_{k=1}^{K} \sum_{i=1}^{L} a_{k,i} p_{k,i} \leq P_{\text{tot}},
\]

\[
\sum_{k=1}^{K} a_{k,i} b_{n,i} \frac{p_{k,i} \xi}{d_{0,n}^m} \leq \frac{I_{\text{thresh}}}{10 Q^{-1}(\rho \sigma_x)}, \quad \forall i, \forall n,
\]
Lagrange Technique

▶ KKT conditions

\[
\begin{align*}
\frac{a_{k,i}\|h_{k,i}\|^2}{N_0 + |h_{k,i}|^2 p_{k,i}} - a_{k,i} b_i \frac{\lambda_i \xi}{d_0} - a_{k,i} \rho_0 &= 0, \\
\lambda_{i,n} \left( \frac{I_{n,i}^{\text{thresh}}}{10^{Q-1}(\rho \sigma_x)} - \sum_{k=1}^{K} a_{k,i} b_i \frac{p_{k,i} \xi}{d_0^\eta} \right) &= 0, \\
\rho_0 \left( P_{tot} - \sum_{k=1}^{K} \sum_{i=1}^{L} a_{k,i} p_{k,i} \right) &= 0.
\end{align*}
\]

▶ Optimal power allocation (knowing the subchannel allocation)

\[
p_{k,i} = \begin{cases} 
\left( \frac{1}{\rho_0} - \frac{N_0}{|h_{k,i}|^2} \right)^+, & \text{if } i \text{ non used by any PU}, \\
\min \left\{ \left( \frac{1}{\rho_0} - \frac{N_0}{|h_{k,i}|^2} \right)^+, \frac{I_{n,i}^{\text{thresh}} d_{0,n_i}^\eta}{10^{Q-1}(\rho \sigma_x) \xi} \right\}, & \text{if } i \text{ used by at least one PU},
\end{cases}
\]

where \( \frac{1}{\rho_0} \) is the water level:

\[
\frac{1}{\rho_0} = \frac{1}{|U|} \left( P_{tot} - \sum_{i \in S} \frac{I_{n,i}^{\text{thresh}} d_{0,n_i}^\eta}{10^{Q-1}(\rho \sigma_x) \xi} + \sum_{i \in U} \frac{N_0}{|h_{k,i}|^2} \right)
\]

\( k_i \) is the secondary user allocated to use the subchannel \( i \).

\( n_i \) is the most primary user affected by the power transmitted by the base station on the subchannel \( i \).
Optimal subchannel and power allocation

1. Select the most sensitive primary user per subchannel

\[ n_i = \arg \min_{n/b_{n,i}=1} \left\{ \frac{d_{0,n_i} \tilde{T}_{n,i}^{\text{thresh}}}{\xi} \right\}, \quad i = 1, \cdots, L \]

2. Select the allocated secondary user per subchannel

\[ k_i = \arg \max_k \frac{|h_{k,i}|^2}{N_o}, \]

3. Run the conventional Cap-limited waterfilling algorithm which solves the simplified problem:

\[
\max \sum_{i=1}^{L} a_{k_i,i} \log_2 \left( 1 + \frac{|h_{k_i,i}|^2 p_{k_i,i}}{N_o} \right)
\]

subject to

\[
\sum_{i=1}^{L} p_{k_i,i} \leq P_{\text{tot}},
\]

\[
p_{k_i,i} \leq \frac{\tilde{T}_{n,i}^{\text{thresh}} d_{0,n_i}^{\eta}}{10^{Q-1}(p^\epsilon \sigma_x) \xi}, \quad \forall i,
\]
Uplink

Problem statement

\[
\max \sum_{k=1}^{K} \sum_{i=1}^{L} a_{k,i} \log_2 \left( 1 + \frac{|h_{k,i}|^2 p_{k,i}}{N_o} \right)
\]

subject to

\[
\sum_{k=1}^{K} a_{k,i} = 1, \forall i,
\]

\[
\sum_{i=1}^{L} a_{k,i} p_{k,i} \leq P_k, \quad \forall k,
\]

\[
\sum_{k=1}^{K} a_{k,i} b_{n,i} \frac{p_{k,i} \xi}{d_{k,n}^\eta} \leq \frac{I_{n,i}^{\text{thresh}}}{10Q^{-1}(p_e \sigma_x)}, \forall i, \forall n,
\]
Lagrange Technique

- **KKT conditions**

\[
\begin{align*}
\frac{a_{k,i}|h_{k,i}|^2}{N_o + |h_{k,i}|^2 p_{k,i}} - a_{k,i} b_{n,i} \frac{\lambda_{i,n} \xi}{d_{k,n}^\eta} - a_{k,i} \rho_k &= 0, \\
\lambda_{i,n} \left( \frac{I_{\text{thresh}}^{n_i}}{10^{Q-1}(p_{\epsilon} \sigma_x)} - \sum_{k=1}^{K} a_{k,i} b_{i,n} \frac{p_{k,i} \xi}{d_{k,n}^\eta} \right) &= 0, \\
\sum_{k=1}^{K} \rho_k \left( P_k - \sum_{i=1}^{L} a_{k,i} p_{k,i} \right) &= 0.
\end{align*}
\]

- **Optimal power allocation** (knowing the subchannel allocation)

\[
p_{k,i} = \begin{cases} 
\left[ \frac{1}{\rho_{k,i}} - \frac{N_o}{|h_{k,i}|^2} \right]^+, & i \in U_c, \\
\min \left\{ \left[ \frac{1}{\rho_{k,i}} - \frac{N_o}{|h_{k,i}|^2} \right]^+, \frac{I_{\text{thresh}}^{n_{k_i,i}} d_{k_i,n_{k_i,i}}^\eta}{10^{Q-1}(p_{\epsilon} \sigma_x) \xi} \right\}, & i \in U_p,
\end{cases}
\]

where \( \frac{1}{\rho_k} \) the water level for user \( k \) determined by:

\[
\frac{1}{\rho_k} = \frac{1}{|U_k|} \left( P_k - \sum_{i \in S_k} \frac{I_{\text{thresh}}^{n_{k_i,i}} d_{k_i,n_{k_i,i}}^\eta}{10^{Q-1}(p_{\epsilon} \sigma_x) \xi} + \sum_{i \in U_k} \frac{N_o}{|h_{k,i}|^2} \right)
\]

\( k_i \) is the secondary user allocated to the subchannel \( i \)

\( n_{k_i,i} \) is the most affected primary base station by the power emitted from the secondary user \( k_i \) on the subchannel...
Proposed Algorithm

- Select most sensitive primary user per subchannel and per secondary user

\[ n_{k,i} = \underset{n/b_{n,i}=1}{\text{arg min}} \left\{ \frac{d_{k,n}^n \tilde{I}_{n,i}^{\text{thresh}}}{\xi} \right\}, \quad \begin{cases} k = 1, \ldots, K. \\ i = 1, \ldots, L. \end{cases} \]

- Impossible consecutive subchannel and power allocation (unlike downlink)

- Perform low cost algorithm to allocate subchannels and power simultaneously
  1. Run the cap-limited waterfilling algorithm over the available subchannels for each user independently.
  2. Compute the capacity for each subchannel
  3. Select the pair \( \{k_i^*, i^*\} \) with the highest capacity,
  4. Allocate the \( i^* \)th subchannel to the user \( k_i^* \)
  5. Update the list of unavailable subchannels
  6. Repeat the above procedure until all the subchannels allocated.
Simulations set up

- Number of subcarriers: $L = 64$
- Number of secondary users: $K = 20$
- Number of primary users: $N = 10$
- Pathloss exponent: $\eta = 3$
- Interference threshold: $I_{th} \in [-130\,dBm, -110\,dBm]$
- Shadowing effect variance:
- Base station power budget: $P_{tot} = 20\,dBm$
- Secondary user power budget: $P_k = 3\,dBm$
- $p_\epsilon = 0.4\%$, $\sigma_x = 5$
Location Information Effect

Figure: Effect of the use of location information instead of channel state information on the capacity of cognitive network.
Proposed Algorithm compared to Exhaustive Search

Figure: Comparison between the total capacity obtained using the proposed algorithm and the exhaustive search algorithm (8 subch, 4 SU, 2 PU).
Impact of the location of the users on the capacity

1. **Scenario 1**: All the SU are located along the circle with a radius of 1 (km) and the PU is located within the cell with the radius of 8 (km).

2. **Scenario 2**: The secondary users as well as the primary user are randomly distributed within the cell. (more practical scenario)

**Figure**: Comparison between the performance of two users distribution scenarios for the downlink case.
Threshold interference impact

Figure: Downlink capacity of various schemes as a function of $p_{\epsilon}$
Presence of inter-carrier interference (ICI)

ICI coefficient between subchannels: \( g_{i,j} = \begin{cases} \frac{\alpha}{(i-j)^2}, & \text{if } i \neq j, \\ 1, & \text{if } i = j, \end{cases} \)

**Downlink**

\[
\sum_{k=1}^{K} \frac{b_i}{d_0^n} \sum_{j=1}^{L} g_{i,j} a_{k,j} p_{k,j} \leq \frac{l_{th}}{10^{Q-1}(p_e \sigma_X)}, \quad \forall i
\]

\[
\sum_{i=1}^{K} \frac{b_i}{d_k^n} \sum_{j=1}^{L} g_{i,j} a_{k,j} p_{k,j} \leq \frac{l_{th}}{10^{Q-1}(p_e \sigma_X)}, \quad \forall i
\]

**Uplink**

\[
\sum_{k=1}^{K} \frac{b_i}{d_0^n} \sum_{j=1}^{L} g_{i,j} a_{k,j} p_{k,j} \leq \frac{l_{th}}{10^{Q-1}(p_e \sigma_X)}, \quad \forall i
\]

\[
\sum_{i=1}^{K} \frac{b_i}{d_k^n} \sum_{j=1}^{L} g_{i,j} a_{k,j} p_{k,j} \leq \frac{l_{th}}{10^{Q-1}(p_e \sigma_X)}, \quad \forall i
\]

\[
p_{k_i,i} = \left[ \frac{d_0^n}{\xi \sum_{j \in U_p} g_{j,i} \lambda_j + \rho_0 d_0^n} - \frac{N_0}{|h_{k,i}|^2} \right]^+, \quad \forall i
\]

\[
p_{k_i,i} = \left[ \frac{d_k^n}{\xi \sum_{j \in U_p} g_{j,i} \lambda_j + \rho_k d_k^n} - \frac{N_0}{|h_{k,i}|^2} \right]^+, \quad \forall i
\]
Impact of inter-carrier interference between subchannels

Figure: Impact of correlation between subchannels on the capacity.
Discrete rate resource allocation

- Adjust power allocated to nearest inferior discrete load
  \[ p'_{ki,i} = \frac{N_0}{|h_{ki,i}|^2} (2^{r_{ki,i}} - 1), \text{ with } r_{ki,i} = \lceil \log_2(1 + \frac{|h_{ki,i}|^2 p_{ki,i}}{N_0}) \rceil \]

- Compute remaining power after readjustment: \( P^- = P_{\text{tot}} - \sum_{i=1}^{L} p'_{ki,i} \)

- Redistribute remaining power using greedy algorithm

1. start from the subchannel with minimal additional power needed to increment bit load
   \[ n_{\text{min}} = \arg \max_{i \in S} |h_{ki,i}|, \text{ where } S = \{i \in \arg \min \{r_{ki,i}\} \} \]

2. compute the power needed to increment its power to next bit
   \[ p^+_{i} = \frac{N_0}{|h_{ki,i}|^2} 2^{r_{ki,i}} \]

3. verify power budget and interference constraints
   \[ i \in U_c \text{ or } (i \in U_p \text{ and } p_{ki,i} + p^+_{i} \leq \frac{I_{\text{thresh}}}{d_{ni,i}^{\eta}} \xi) \]
   \[ p_{ki,i} \leq P^- \]

4. update remaining power and search next subchannel
   \[ P^- = P^- - p_{ki,i} \]
Discretization effect on the performances in Downlink

Figure: Effect of discrete rate allocation on the performance of the cognitive network for downlink.
Collocated subchannels allocation

1. Construct a capacity matrix \( C = \{c_{k,i}\}^{K \times L} \), and a validity indication matrix \( V = \{v_{k,i}\}^{K \times L} \) and initialize it as valid.
2. Run an individual cap-limited waterfilling for each user.
3. Compute the elements of \( C \) using \( c_{k,i} = \log_2(1 + \frac{|h_{k,i}|^2 p_{k,i}}{N_0}) \).
4. Find the element with the highest capacity among the valid elements:
   \[ \{k^*, i^*\} = \arg \max_{k,i} v_{k,i} c_{k,i} \]
5. Check if the user \( k^* \) already has other allocated subchannel(s). If so, go to 6, otherwise, proceed to 7.
6. Check if the subchannel \( i^* \) is adjacent to the already allocated subchannels for the user \( k^* \).
   - If so, proceed to 7,
   - Otherwise, the subchannel \( i^* \) can not be allocated to the user \( k^* \): So mark it \( i^* \) as invalid and go back to 4.
7. Allocate the subchannel \( i^* \) to the user \( k^* \) and mark it as invalid for all other users.
8. Check the surrounding (left and right) elements, if they are invalid change them as valid.
9. Go back to 2 and repeat until all the subchannels are allocated.
## Comparison between different scenarios

<table>
<thead>
<tr>
<th>subchannel</th>
<th>power</th>
<th>rate</th>
<th>Up / Downlink</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>no restriction</td>
<td>waterfilling</td>
<td>fractional</td>
</tr>
<tr>
<td>2</td>
<td>no restriction</td>
<td>waterfilling</td>
<td>discrete</td>
</tr>
<tr>
<td>3</td>
<td>no restriction</td>
<td>fixed power</td>
<td>fractional</td>
</tr>
<tr>
<td>4</td>
<td>collocated</td>
<td>waterfilling</td>
<td>fractional</td>
</tr>
<tr>
<td>5</td>
<td>collocated</td>
<td>waterfilling</td>
<td>discrete</td>
</tr>
</tbody>
</table>
Figure: Effect of discrete rate and allocation of collocated subchannels on the performance of the cognitive network.
Conclusion

▶ The use of Location Information is more practical for resource allocation in Cognitive Radios than the Channel State Information.

▶ The proposed algorithms are low-cost and optimal for downlink and suboptimal for uplink.

▶ The proposed model is valid for multiple primary user networks with different thresholds of interference per subchannel.

▶ The suggested scenarios allow easier implementations:
  ▶ Integer bit loading
  ▶ Collocated subchannel allocation

Future work

▶ Multiple secondary networks

▶ Pricing: introduce prices of sharing subchannels.
Thank you for your attention
Questions ??
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