DISTORTION CONTROL FOR PACKET-ERASURE CHANNELS

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ABSTRACT

We investigate the problem of finding minimum-distortion policies for streaming delay-sensitive but distortion-tolerant data over a packet-erasure channel. We consider cross-layer approaches which exploit the coupling between presentation and transport layers. We make the natural assumption that the distortion function is convex and decreasing, and we show that the optimum open-loop transmission policy is independent of the form of the distortion function. We then find a computationally efficient closed-loop heuristic policy which outperforms the open-loop policy and performs virtually as well as the optimum closed-loop policy.

Index Terms— Distortion, source-coding, scheduling

1. INTRODUCTION

In communication networks, traditionally, the source encoding is done independently of the network conditions. In the classical network architecture, the source symbols are encoded in the presentation layer, while the transport layer takes care of providing errorfree transmission by the use of channel coding or retransmissions. In the case of packet-erasure channels, packets traveling through the network are dropped randomly depending on the channel condition. When immediate error-free feedback is available, the best one can do is to retransmit each dropped packet repeatedly until it reaches its destination. When dealing with delay-sensitive applications with a hard deadline for every packet, this approach can be modified to one which repeats the transmission of each lost packet until either the packet is expired or it has reached its destination. However, when the transmitted data is distortion-tolerant, the overall distortion of the received message can significantly be decreased by calculatedly sacrificing less significant bits of one packet for more significant bits of another.

We consider the problem of transmitting a finite set of delay-sensitive source symbols. This is sometimes referred to as “streaming” and is used in applications such as video-on-demand where a server pre-stores encoded media and transmits it on demand to a client for playback in real time.

We study the distortion-delay tradeoff for a packet-erasure channel by considering a source-destination pair connected through a single-link, packet-erasure channel, as shown in Figure 1. A number of source symbols residing at the source are to be encoded and transmitted to the destination before their corresponding deadlines. Each reconstructed symbol will result in a distortion which is a decreasing, convex function of the number of its bits received. Our goal is to find a transmission policy which minimizes the total expected distortion. A policy determines what bits of what symbol to transmit at any time, based on the state of the system at that time. Finding the optimum policy depends on the values of the distortion function and, except for special trivial cases, can be computationally very costly. In this paper, we first find an optimum open-loop policy and then propose computationally inexpensive suboptimal policies which have near optimal performance, as we show through numerical analysis.

The problem of rate-distortion optimized streaming of layered video has been addressed under various scenarios in the literature. To the best of our knowledge the most related works to the one we are presenting have been carried out in [1], [2], and [3]. Miao and Ortega [1] propose a low-complexity heuristic algorithm for scheduling of packet transmission. However, the scope of their results is limited to the case where the number of layers representing each symbol is predetermined. Podolsky et al. [2] use a Markov chain analysis to find the optimal policy for transmitting layered
media at a fixed rate over a lossy channel. However, since the state space grows exponentially with the size of the parameter space, no general solution is presented in that paper. In [3], an optimum solution to the streaming problem over an error-free channel is found, however, no general results are presented for the case of packet-erasure channels. A brief survey of other related problems can be found in [4]. We leave a full survey of the related literature to future correspondences due to space limitations.

2. PROBLEM FORMULATION AND NOTATION

Consider a source-destination pair connected through a single-link, packet-erasure channel as shown in Figure 1. $N$ source symbols are residing at the source and are to be encoded and transmitted to the destination before their deadlines $M_1 \leq M_2 \leq \ldots \leq M_N$. The time is slotted and at every time slot, $B$ bits of information can be transmitted over the link. Each $B$-bit packet will either reach the destination in its entirety with probability $p$, or will be entirely lost otherwise. We make the simplifying assumption that the $B$ bits transmitted at each time slot must correspond to a single symbol. Once we make this assumption, without loss of generality, we can assume that $B = 1$.

At each time slot $t$, let $b(t)$ be an $N$-vector whose $i$th element, $b_i(t)$, is the number of bits corresponding to the $i$th symbol successfully received by the beginning of time slot $t$. Therefore, $b(t)$ indicates the state of the system at time $t$. Let $s(t)$ be an $N$-vector whose $i$th element, $s_i(t)$, is the number of bits corresponding to the $i$th symbol transmitted in time slot $t$. If the transmission at time slot $t$ is successful, we get $b(t + 1) = b(t) + s(t)$. Since $B = 1$, $s(t)$ is always a unit vector with all but one of its elements set to zero. We denote the transmission policy by the function $\phi(\cdot, \cdot)$ such that $s(t) = \phi(b(t), t)$. We wish to find a policy $\phi$ which minimizes the total expected distortion while meeting the deadline constraints. In other words, $\phi(b(t), t) = \phi(t)$, for some function $\phi(\cdot)$. Therefore, we only need to decide on $z_i(T)$, the total number of bits corresponding to every source symbol to transmit by the end of the session. In this section we use $b_i$ and $z_i$ in place of $b_i(T)$ and $z_i(T)$. Note that $b_i$ is a binomial($z_i, p$) random variable, therefore, $E[d(b_i)]$ is a function of $z_i$. If we let $g(z_i) = E[d(b_i)]$, the problem statement simplifies to finding the $N$-vector $z$ that minimizes $G(z) = \sum_{i=1}^{N} g(z_i)$, and satisfies (1) and (2). We refer to this problem as $P_{OL}$. The following algorithm finds an optimum solution to $P_{OL}$.

**Open-Loop Algorithm**

1. Let $\hat{j} = \max \left\{ \arg \min_j \left\{ \left\lfloor \frac{M_j}{T} \right\rfloor \right\}_{j=1}^{N} \right\}$, $k = M_j - \hat{j} \left\lfloor \frac{M_j}{T} \right\rfloor$

2. Set $z^*_j = \left\{ \left\lfloor \frac{M_j}{T} \right\rfloor, j = 1, \ldots, j - k \right\} + 1$, $j = \hat{j} - k + 1, \ldots, \hat{j}$

3. If $\hat{j} < N$, remove $\{M_j\}_{j=1}^{\hat{j}}$, set $j = j - \hat{j}$ for $j > \hat{j}$, update the remaining $M_j$’s, and go back to step 1. Stop otherwise.

**Theorem 1** The integer-valued vector $z^*$ found by the open-loop algorithm solves $P_{OL}$ for any convex and decreasing function $d(\cdot)$.

**Proof** To prove this, we first show that for a decreasing and strictly convex $d(\cdot)$, the function $g(\cdot)$ is also decreasing and has increasing forward differences. This could be seen as an equivalent of convexity for the discrete function $g(\cdot)$. Using this property, we prove that for a vector $z$ to solve $P_{OL}$, we must have

$$\sum_{i=1}^{j} z_i = M_j$$

where $j = \max \left\{ \arg \min_j \left\{ \left\lfloor \frac{M_j}{T} \right\rfloor \right\}_{j=1}^{N} \right\}$. We next show that among all integer-valued vectors which meet (3), $z^*$ results in the smallest $G(z)$. We then show that $z^*$ meets (1) and (2) and therefore is feasible, and hence it solves $P_{OL}$ for a strictly convex $d(\cdot)$. We then use a continuity argument to extend this result for the case where $d(\cdot)$ is merely convex. We skip the details of the proof due to space limitation.

Note that $z^*$ does not depend on the form of the distortion function and solves (although not necessarily uniquely) $P_{OL}$ for any convex and decreasing function $d(\cdot)$.

4. SUBOPTIMAL CLOSED-LOOP POLICY

In this section we present a closed-loop algorithm that is computationally inexpensive and improves the performance compared to the optimal open-loop policy. In order to do this, we employ the idea of Certainty Equivalent Controllers [5]. The certainty equivalent controller is a suboptimal control scheme that applies, at each stage, the action that would be optimal if the random quantities were fixed at some “typical” value. The way we apply this to our problem is to find at each time
We refer to this problem as algorithm finds an actual optimum policy depends on the form of the rate-distortion packet-erasure channels. We proposed different transmission and significantly outperform the open-loop policy and perform very close to optimal. As we can see, the CEC policies showed through numerical evaluation that the CEC policy proposed, results in an average distortion very close to the minimum distortion. Another advantage of the CEC policy is that it can be used for queuing systems with random arrivals, since it does not require for the entire data to be available at the source at the beginning of transmission. Finally, a natural extension of this line of research can be carried out into a network coding framework by considering the distortion as the performance criterion as opposed to the traditional throughput criterion.

6. REFERENCES


5. CONCLUSION

We studied optimum streaming of delay-sensitive data over packet-erasure channels. We proposed different transmission policies for minimizing the expected distortion. While the actual optimum policy depends on the form of the rate-distortion function and is generally computationally costly, our proposed policies are independent of the form of the distortion function and are computationally inexpensive. We furthermore showed through numerical evaluation that the CEC policy proposed, results in an average distortion very close to the minimum distortion.

Figure 2 depicts a comparison between the performance of the open-loop policy and the CEC policy with three different heuristics for finding \( s(t) \) for the case where \( N = 4 \) and \( T = 10 \). We have plotted the average expected difference between each policy and the optimum solution, which we find by exhaustive search. As we can see, the CEC policies significantly outperform the open-loop policy and perform very close to optimal.

![Figure 2](image-url)