DOE
DESIGN OF EXPERIMENT
EXPERIMENTAL DESIGN
Lesson 5
Framework of lesson 5:

• Introduction to the project $2^k$ (2 levels, k factors)
• Calculation of main effects and interactions in the case of only two factors
• Generalization to the case with three or more factors: the table of the signs
• The modern notation and order of Yates
• The algorithm of Yates
• Examples and critical analysis on the method
PROJECT $2^k$ vs. Greek-Latin Squares

- Problems with many factors (combinations = levels$^\text{factors}$)
  - Greek-Latin Squares
  - Project $2^k$ and $2^{k-p}$

- Disadvantages:
  - Disregard the interaction effects: if this assumption proves wrong, it would compromise the estimated effects of factors and their evaluation of significance.

- Advantages:
  - The number of combinations for which to carry out the tests is very reduced (an analysis of many factors is always reduced to a plane to two factors alone), while maintaining any number of levels
PROJECT $2^k$ vs. Greek-Latin Squares

• Problems with many factors (combinations = levels$^\text{factors}$)
  • Greek-Latin Squares
  • Project $2^k$ and $2^{k-p}$

• Disadvantages:
  • The number of layers is limited to two, often assuming a linear dependence of the dependent variable factors in play, is not always likely

• Advantages:
  • It is possible to assess all interactions, not only between two factors, but also among all those involved. The approach is compatible with techniques that allow to further reduce the combinations to be examined
Suppose that a survey has been designed to evaluate the statistical influence on the volume of sales of two factors: the price and location of the product on the shelf.

<table>
<thead>
<tr>
<th></th>
<th>Low Level</th>
<th>High Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factor A:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>positioning</td>
<td>Lower shelf</td>
<td>Higher shelf</td>
</tr>
<tr>
<td>Factor B:</td>
<td></td>
<td>High Level</td>
</tr>
<tr>
<td>cost</td>
<td>10€</td>
<td>13€</td>
</tr>
</tbody>
</table>

It's purely conventional notation, but once levels were indicated as high and low, these should be kept, to avoid an error.
• Treatment combinations are 4 \((2^2)\):
  • low level \(\rightarrow\) by convention indicated as 0
  • High level \(\rightarrow\) by convention indicated as 1
  • \(a_0b_0\): A low level, B low level
  • \(a_1b_0\): A High level, B low level
  • \(a_0b_1\): A low level, B High level
  • \(a_1b_1\): A High level, B High level

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td>(a_0b_0)</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>(a_1b_0)</td>
</tr>
</tbody>
</table>
PROJECT 2^2

• Estimation of the effect of A:

\[ a_1b_1 - a_0b_1 \]  
Estimation of the effect of A on B high

\[ a_1b_0 - a_0b_0 \]  
Estimation of the effect of A on B low

\[ \text{Sum}/2 \]  
Estimation of the effect of A (on any level of B)

• Estimation of the effect of B:

\[ a_1b_1 - a_1b_0 \]  
Estimation of the effect of B on A high

\[ a_0b_1 - a_0b_0 \]  
Estimation of the effect of B on A low

\[ \text{Sum}/2 \]  
Estimation of the effect of B (on any level of A)
Estimation of the effect of B over A:

\[ a_1b_1 - a_0b_1 \]  
Estimation of the effect of A on B high

\[ a_1b_0 - a_0b_0 \]  
Estimation of the effect of A on B low

\[ \text{difference/2} \]  
Estimation of the effect of B over the effect of A

Estimation of the effect of A over B:

\[ a_1b_1 - a_1b_0 \]  
Estimation of the effect of B on A high

\[ a_0b_1 - a_0b_0 \]  
Estimation of the effect of B on A low

\[ \text{difference/2} \]  
Estimation of the effect of A over the effect of B

By definition the interaction of A to B is equal to the interaction of B to A. Thus, the interaction term is unique.
• It is therefore indifferent to say that the relation of dependence of the response with respect to A in turn depends on B, or that the dependence of the variable to B depends in turn by A.
• It should be noted that for the determination of the effects is necessary to know all four of the results, corresponding to each of the treatment combinations. By combining these results, it is possible to assess the effects, i.e. how many tc – 1 (treatment conditions – 1). These effects (two main effects directly attributable to the factors A and B) and an interaction AB are evaluated separately with respect to one another.
• This is a big advantage of the full factorial plan, which is lost when further technical limitation of combinations are used (fractional factorial)


**PROJECT 2**

• Usually tables of signs are used to compute the effects:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>AB</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a_0b_0)</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>(a_1b_0)</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>(a_0b_1)</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>(a_1b_1)</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

• It is then required to divide by 2 (in general by \(2^{k-1}\)).
• For example for the effect of A:
  • \(A = (-a_0b_0 + a_1b_0 - a_0b_1 + a_1b_1)/2\)
• The two treatments at high level of A are compared (we therefore perform a difference) to the two treatments at low level.
PROJECT 2²

• Usually tables of signs are used to compute the effects:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>AB</th>
</tr>
</thead>
<tbody>
<tr>
<td>a₀b₀</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>a₁b₀</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>a₀b₁</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>a₁b₁</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

• In the evaluation of A signs are alternated, in the one of B they are alternated two by two, in the one of the iteration they are given by the products of the signs of the factors interacting.

• Rules of orthogonality and balance are respected.
PROJECT 2^2

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>AB</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>2.5</td>
<td>2.5</td>
<td>-2.5</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>-3</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Analysis of the 4 cases:

1. In this case, the effect of A does not vary with the levels of B, and so the effect of B is the same at the two levels of A. It therefore have the two main effects, but the absence of interaction.

2. The effect of A is different at the level 0 and level 1 of B. In the first case this effect is +5, in the second 0. The same applies to the factor B. On average, the two effects are \((5 + 0) / 2 = 2.5\). This example shown how an experiment that make vary one factor at a time can lead to errors of evaluation: the effect of A to B lower is +5, that of B to A lower is also +5, but the sum of the two is +5 and not +10 as you would expect.

3. The effects of A are +3 at B low and -3 at B high, so the overall effect is null. The same for B. In this case the effect is all on the interaction. It is also known as, in the presence of strong interactions, the principal effects are meaningless.

4. The values are all the same: the effects are not null. However, null effect does not mean null result.
PROJECT 2²

• The effects calculated are insensitive to linear operations. If all the results were to be varied by a constant, the effects would not change. This is because the effects are by definition comparisons, i.e. differences and in differences the constants are removed.

• If you change units, all the results are multiplied by a constant. Also in this case the effect does not vary. The new unit will be numerically different, but the physical entity remains the same.

• Consider the example that relates the life of a battery with the brand and the device in which it is used. If the duration, initially expressed in hours, is converted in minutes, all the results are multiplied by 60, and so the effects. The temporal duration, however, remains obviously the same.
• Usually the treatment combinations are indicated by reference to the Yates contracted notation.

To switch from traditional notation to that of Yates, you must bring the subscript (0 or 1) to the exponent, then calculating the power and the product literal. therefore:

- $a_0b_0 \rightarrow a^0b^0 = 1$;
- $a_1b_0 \rightarrow a^1b^0 = a$;
- $a_0b_1 \rightarrow a^0b^1 = b$;
- $a_1b_1 \rightarrow a^1b^1 \rightarrow ab$

• The treatment combinations can be tried in any order, but in the analysis of the effects the data is arranged in a precise order. Each letter is followed by the combinations of this with all the letters already entered. The new letters are placed in alphabetical order. For a plan $2^2$ the order is: 1 a b ab
PROJECT 2³

- Three factors (A, B, C) with two level each

<table>
<thead>
<tr>
<th>Traditional</th>
<th>a₀b₀c₀</th>
<th>a₁b₀c₀</th>
<th>a₀b₁c₀</th>
<th>a₁b₁c₀</th>
<th>a₀b₀c₁</th>
<th>a₁b₀c₁</th>
<th>a₀b₁c₁</th>
<th>a₁b₁c₁</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yates</td>
<td>1</td>
<td>a</td>
<td>b</td>
<td>ab</td>
<td>c</td>
<td>ac</td>
<td>bc</td>
<td>abc</td>
</tr>
</tbody>
</table>

Estimation of the effect of A:

- a – 1  Estimation of the effect of A with B and C low
- ab – b Estimation of the effect of A with B high and C low
- ac – c Estimation of the effect of A with B low and C high
- abc – bc Estimation of the effect of A with B e C alti
- sum/4   Estimation of the effect of A (averaged over all the levels of B and C)
PROJECT 2³

• Estimation of the effect of the interaction AB:

abc – bc  Estimation of the effect of A with B and C high
ac – c  Estimation of the effect of A with B low and C high
ab – b  Estimation of the effect of A with B high and C low
a – 1  Estimation of the effect of A with B and C alti

sum/4  Estimation of the effect of interaction AB (averaged over all the levels of C)
• Estimation of the effect of the interaction among the three factors: ABC
• It is a measure of how much the interaction AB varies varying the levels of C.

\[
\begin{align*}
\text{abc} - \text{bc} & \quad \text{Estimation of the effect of A with B and C high} \\
\text{ac} - \text{c} & \quad \text{Estimation of the effect of A with B low and C high} \\
\text{ab} - \text{b} & \quad \text{Estimation of the effect of A with B high and C low} \\
\text{a} - 1 & \quad \text{Estimation of the effect of A with B e C low} \\
\text{sum}/4 & \quad \text{Estimation of the effect of interaction ABC}
\end{align*}
\]
ESTIMATION OF THE EFFECT OF INTERACTION AMONG THE 3 FACTORS ABC:

It is a term that takes into account:

• How much the interaction AB varies varying the levels of C
• How much the interaction AC varies varying the levels of B
• How much the interaction BC varies varying the levels of A
• The three interpretations have similar meaning. In any case the term ABC is unique.
The calculation of the effects can be eased by the use of the table of signs:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>AB</th>
<th>C</th>
<th>AC</th>
<th>BC</th>
<th>ABC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>a</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>b</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>ab</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>c</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>ac</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>bc</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>abc</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

- Principal effects: alternated signs at group of 1 ($2^0$), 2($2^1$), 4 ($2^2$), ...
- Interaction effects: the signs are given by the products of the signs of the interacting factors
Suppose that you have set up a statistical survey to assess the impact on the volume of sales of a product by three factors: the placement on the shelves, the price and the size of the box (with the same content).

<table>
<thead>
<tr>
<th>Factor</th>
<th>Low Level</th>
<th>High Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factor A: positioning</td>
<td>Lower shelf</td>
<td>Higher shelf</td>
</tr>
<tr>
<td>Factor B: price</td>
<td>10€</td>
<td>13€</td>
</tr>
<tr>
<td>Factor C: dimension of the box</td>
<td>compact</td>
<td>big</td>
</tr>
</tbody>
</table>
A = principal effect of positioning = 0.0125
B = principal effect of price = -0.0485
AB = interaction between positioning and price = -0.0060
C = principal effect of the dim. of the box = 0.0060
AC = interaction between positioning and dim. box = -0.0015
BC = interaction between price and dim. box = 0.0045
ABC = interaction among all the factors = -0.0050
The higher effect in absolute value is B, which is the main effect linked to the price. Logically, a price increase lowers sales volume. As it regards the other main effects, a high shelf placement increases sales and also a positive effect, although limited, has given by increasing the size of the box.

Interaction AB is negative, indicating that an increase in the price from 10 € to 13 € reduces the positive effect of increasing the level of the shelf. It can also be interpreted by observing that the placement on the top shelf makes it even more negative the effect of the price.

Interaction BC is positive: increasing the price increases the positive effect related to the size of the box. Increasing the size, mitigates the negative effect on the price.
The interaction between all three factors (ABC) is negative, -0.0050. They can be given different interpretations, all equally valid. Increasing the level of C, increased dimension of the box, made even more negative the interaction AB. Increasing the level of B, that is the higher price, the positive interaction AC (between positioning and size of the box) is reduced. Finally, the increase of the level of A, that is moving the product from the lower to the higher shelf, the positive interaction BC is reduced.
PROJECT $2^k$

- When $k$ factors are in play, it is usual to speak of projects $2^k$, i.e. with $k$ factors each with two levels. The total number of combinations is clearly $2^k$, while the number of effects, principals and of interaction between two or more factors, is given by $2^k-1$. For example, if $k = 2$, there are 4 combinations ($1, a, b, ab$) and 3 effects ($A, B, AB$), if $k = 3$, there are 8 combinations ($1, a, b, ab, c, ac, bc, abc$) and 7 effects ($A, B, AB, C, AC, BC, ABC$).

- For the calculation of the effects you can use the tables of signs. These are tabulated, or obtainable according to the rules already seen: on the column of the main effects the signs are interleaved in groups, 1, 2, 4, 8, ..., $2^{m-1}$ (for the $m$-th principal effect). The signs for the interaction effects are obtained easily, making the product of the signs on the columns related to the effects of the main factors interacting.
YATES ALGORITHM

• Derive the tables of the signs for many factors and apply them is not immediate. Therefore it is often convenient to use an algorithm for systematic calculation of the effects in plans $2^k$. It was developed by Yates in 1937.
• It is very simple (only consists of addition and subtraction) and can be automatized by any software.
• It is of a general nature and must be applied in cascade $k$ times, i.e. as many as the factors.
• The first step is to write in column the results in the order of Yates (this is for example the average of the n measurements for each treatment combination).
The values in the third column are proportional to the estimation of the effects:
YATES ALGORITHM: EXAMPLE $2^3$

Results

Step 1

Step 2

K-th Step

1
a
b
ab
c
ac
bc
abc
...

$2^k \mu$
$2^{k-1} A$
$2^{k-1} B$
$2^{k-1} AB$
$2^{k-1} C$
$2^{k-1} AC$
$2^{k-1} BC$
$2^{k-1} ABC$
...

...
YATES ALGORITHM: EXAMPLE $2^3$

- It is the only case in which you have a perfect connection between the tiny notation (combinations of various factors at different levels) and the capital (effects of the factors and their interactions).
- On the first line you add up all contributions obtaining the (estimated) Grand Mean multiplied by the number of results.
- To find (estimation of) the effects you must divide each value calculated by $2^{k-1}$. 
YATES ALGORITHM: EXAMPLE 2³

In order to have the aforementioned correspondence between the indication of the treatment combinations and the effects, it is essential to respect the order of Yates. Otherwise the algorithm can not work.

<table>
<thead>
<tr>
<th>Yates</th>
<th>Result</th>
<th>Step 1</th>
<th>Step 2</th>
<th>Step 3</th>
<th>Estimation of...</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.062</td>
<td>0.136</td>
<td>0.166</td>
<td>0.356</td>
<td>8μ</td>
</tr>
<tr>
<td>a</td>
<td>0.074</td>
<td>0.030</td>
<td>0.190</td>
<td>0.050</td>
<td>4A</td>
</tr>
<tr>
<td>b</td>
<td>0.010</td>
<td>0.139</td>
<td>0.022</td>
<td>-0.194</td>
<td>4B</td>
</tr>
<tr>
<td>ab</td>
<td>0.020</td>
<td>0.051</td>
<td>0.028</td>
<td>-0.024</td>
<td>4AB</td>
</tr>
<tr>
<td>c</td>
<td>0.057</td>
<td>0.012</td>
<td>-0.106</td>
<td>0.024</td>
<td>4C</td>
</tr>
<tr>
<td>ac</td>
<td>0.082</td>
<td>0.010</td>
<td>-0.088</td>
<td>0.006</td>
<td>4AC</td>
</tr>
<tr>
<td>bc</td>
<td>0.024</td>
<td>0.025</td>
<td>-0.002</td>
<td>0.018</td>
<td>4BC</td>
</tr>
<tr>
<td>abc</td>
<td>0.027</td>
<td>0.003</td>
<td>-0.022</td>
<td>-0.020</td>
<td>4ABC</td>
</tr>
</tbody>
</table>
$2^k$ WITH REPETITIONS

• The factorial design is obviously also possible in the presence of repetitions. For each combination of treatments will obtain different results based on which to calculate the effects. The calculation of these is addressed in the same way, whether using the table of the signs or the algorithm of Yates.

• Due to the linearity of the operations performed, it is indifferent to compute the average results for each combination and find the effects on the base of the averages, or find the effects for each outcome and then make the average of the effects.
PRINCIPAL EFFECTS VS. INTERACTIONS

• We must reflect on the fact that the estimation of the principal effects should be taken with great caution in presence of strong interactions.
• Let’s suppose to have only two factors, A e B.
• The effect of A is given by the average of the effect at high B and the effect with low B.
• The interaction AB is related to the difference between the effect of A at high B and the effect of A at low B.

\[
\begin{array}{c|c|c}
A & B & \text{Effect} \\
\hline
0 & 0 & 10 \\
0 & 1 & 13 \\
1 & 0 & 13 \\
1 & 1 & 16 \\
\end{array}
\]

\[
A = (3 + 3)/2 = 3 \\
AB = (3 - 3)/2 = 0
\]

\[
\begin{array}{c|c|c}
A & B & \text{Effect} \\
\hline
0 & 0 & 10 \\
0 & 1 & 13 \\
1 & 0 & 13 \\
1 & 1 & 10 \\
\end{array}
\]

\[
A = (-3 + 3)/2 = 0 \\
AB = (-3 - 3)/2 = -3
\]
PRINCIPAL EFFECTS VS. INTERACTIONS

Therefore if these two effects (at high and low B) are equal, there is no interaction. If they are different, there is a term that takes into account how the effect of A depends on the levels of B. If this term is very high, indicates that there is strong dependence, i.e. that the effect of A with high B is very different from that with low B. Since the effect of A is given by the total average of two very different quantities, the result may not be very reliable.

$$A = \frac{(3 + 3)}{2} = 3$$
$$AB = \frac{(3 - 3)}{2} = 0$$

$$A = \frac{(-3 + 3)}{2} = 0$$
$$AB = \frac{(-3 - 3)}{2} = -3$$
EXAMPLE: STRONG INTERACTION

Impact over the appreciation of cigarettes on the basis of the name chosen for the brand and the sex of the users

According to current notations:

<table>
<thead>
<tr>
<th></th>
<th>Low Level</th>
<th>High Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factor A: sex</td>
<td>Male</td>
<td>Female</td>
</tr>
<tr>
<td>Factor B: brand</td>
<td>Frontiersman</td>
<td>April</td>
</tr>
</tbody>
</table>
EXAMPLE: STRONG INTERACTION

With Yates notation:
1 = 4.44; a = 2.04; b = 3.50; ab = 4.52
The effects are:
A = -0.69; B = +0.77; AB = 1.71

As already noted, there is a strong interaction. The effect of A is negative, which indicates that men (low level) appreciate more the two brands of cigarettes. The effect of B is instead positive, which means that the mark April is the most appreciate. Strong positive interaction means that the switch from males to females increases much appreciation of April. In the same way the passage from Frontierman to April improved appreciation of women (the effect goes from negative to positive).
EXAMPLE: STRONG INTERACTION

- Calculating the effects separately, the most interesting is the B:
  - Effect of B with high A (for women): $4.52 - 2.04 = 2.48$
  - Effect of B with low A (for men): $3.50 - 4.44 = -0.94$
- The average would be 0.77 (which would indicate a greater appreciation for April), but the phenomenon is best described by the two separate effects previously calculated.
- To perform these two estimations are sufficient only two of the four values of the complete factorial plan.
- In strong interactions, the principal effects (global) should be taken with certain precautions and may be useful to utilize the calculation of the partial effects.
In a plan $2^k$ we consider only two levels. If there are only two levels (e.g. male or female), the problem does not arise. For continuous variables we must choose two appropriate levels. The effects may depend on the levels chosen? In case of strong nonlinearity unfortunately yes. Assume to study the elasticity of the skin as a function of pressure and temperature. The distribution is not linear with the point of maximum for a given temperature. We obtained curves for different pressures.
LEVELS OF THE FACTORS

Elasticity (%)

<table>
<thead>
<tr>
<th>For</th>
<th>Effect of $p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t=t_1$</td>
<td>$22 - 30 = -8$</td>
</tr>
<tr>
<td>$t=t_2$</td>
<td>$29 - 58 = -29$</td>
</tr>
<tr>
<td>$t=t_3$</td>
<td>$47 - 20 = +27$</td>
</tr>
<tr>
<td>$t=t_4$</td>
<td>$18 - 15 = +3$</td>
</tr>
</tbody>
</table>
# Levels of the Factors

<table>
<thead>
<tr>
<th>Levels ((t_1, t_2))</th>
<th>Levels ((t_3, t_4))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P = (-8-29)/2 = -18.5)</td>
<td>(P = (27+3)/2 = +15)</td>
</tr>
<tr>
<td>(T = ((29-22) + (58-30))/2 = +17.5)</td>
<td>(T = ((18-47) + (15-20))/2 = -17)</td>
</tr>
<tr>
<td>(PT = (-29-(-8))/2 = -10.5)</td>
<td>(PT = (3-27)/2 = -12)</td>
</tr>
</tbody>
</table>
LEVELS OF THE FACTORS

• The effects depend heavily on the levels chosen. It is only when the response curves are linear and parallel that the effects can be studied properly regardless of the levels chosen.

• Then you get into a vicious circle, because to properly plan the experiment, we need to select adequate levels and to set the levels it is necessary to have an idea of the trend of the response.

• To get this information an experimental control is needed. Then you need an experiment to be able to better organize the analysis. We come out of the vicious circle, relying on literature or experience and/or performing the preliminary experiments with exploratory purpose.
The project $2^k$ is often wrongly associated with the concept of a multitude of experiments with great waste of resources. The typical counterpart is the plan that provides to vary one factor at a time, fixing the others.
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Factores one by one: 2 values for each combination.
Comparison between the two project strategies:

1. In the case of the "one factor at a time" you will have a total of six results against four of the project $2^k$. So on one hand the draft "a factor at a time" is more expensive but on the other hand the number of results used to evaluate the major effects is same as the $2^k$, that is to say 4. In response to increased use of resources, reliability estimates of the effect is still the same.

2. The "one factor at a time" does not allow to evaluate the effects of interaction. This is a serious defect because in general can not be excluded a priori the absence of interactions.
Comparison between the two project strategies:

3. In the factorial design each main effect is obtained by averaging the levels of the other factors: for example the effect of A is the average of the effects of high and low B. In the case of the project "one factor at a time" instead the effect of A is evaluated only at low B and the effect of B only at low A. In the presence of strong interaction, the main effect of an overall factor is often meaningless and have more sense the effects (for example of A) at the two levels of the other factor (i.e. B). In this case the project "one factor at a time" does not allow to assess the interaction, provides more data (4 vs. 2) to estimate for example the effect of A at low-B, however, it is not possible an estimate of the effect of A at high B.
Comparison between the two project strategies:

4. The project "one factor at a time" allows to realize the effects of other factors, which may pollute the results by contributing experimental uncertainty (error). Suffice is to evaluate the differences between the two results for each of the three combinations studied. However, a similar analysis would be possible even with a factorial design with repetitions, but the number of required data would be higher (by 4 would increase to 8) and so would be the costs.

In conclusion, the factorial design is by far the better, both for the possibility of estimating the interactions, and for the one to estimate more reliably the principal effects.