DOE
DESIGN OF EXPERIMENT
EXPERIMENTAL DESIGN

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Framework of lesson 1:

- The concept of experiment
- The role of the DOE: its usefulness and essential aspects
- Practical example in the use of DOE
- Design of a monofactorial experiment
- The statistical model and the square sum
- Analysis of variance (ANOVA) and the Fisher ratio
- Practical Examples
Experimentation

Physical phenomena

FACTORS

LEVELS

Variable 1 $X_1$

Variable 2 $X_2$

... $X_n$

Value 1

Value 2

... $X_n$

Value 1

Value 2

... $X_n$

Value 1

Value 2

... $X_n$
Experimentation

Physical phenomena

FACTORS

LEVELS

Output Variables
$Y_1, Y_2, ..., Y_m$

Independent Variables
$x_1, x_2, ..., x_n$

Dependent Variables
$Y_1, Y_2, ..., Y_m$
Experimentation and DOE

- Role of DOE
  - Which Factors must be studied?
  - Which levels must be considered?
  - How to combine factors and levels?

**RELATION?**

Independent Variables

$x_1, x_2, ..., x_n$

Dependent Variables

$Y_1, Y_2, ..., Y_m$
Experimentation and DOE

- Determination of factors that might affect outputs
- Determination of number of level per each factor
- Identify the number of repetition
- Plan of tests
- Execution of the experiment
- Statistical analysis and individuation of significant effects

A → 3 levels
B → 5 levels
C → 8 levels
Example

• PROBLEM

• Characterization of the friction behaviour of plastic films for cigarettes packaging

Packet:
dimensions:
80 mm x 60 mm x 20 mm

BOPP polipropilene film a double orientation
FACTORS

• Research factors having a potential impact over the process.
  a) Physical and chemical properties of the film
  b) Characteristics of the sliding surface
  c) Temperature
  d) Sliding speed
  e) Pressure at the packet-film interface
LEVELS

• a) 15 films from different vendors
• b) 4 different surfaces

• c) 3 levels of temperature (22°C ÷ 150°C)

• d) 3 levels of sliding speeds (150 ÷ 1200 mm/min)
• e) 3 levels of interface pressure
Experiments

Test for friction coefficient evaluation

- Dependent variable: friction coefficient
TESTS COMBINATIONS

- Total number of combinations (all the factors and all the levels) \(\rightarrow 15 \cdot 4 \cdot 3 \cdot 3 \cdot 3 = 1620\)
- 3 repetition \(\rightarrow 4860\) experiments!!
  - 15 min per experiment \(\rightarrow 151\) working days

FIRST PHASE

- Screening phase for experiments reduction \(\rightarrow\) fractional factorial design (Taguchi) \(\rightarrow 2^{k-p} (2^{5-1})\)
- Only 48 tests \((3 \cdot 2^{5-1})\) \(\rightarrow\) Identification of significant factors (ANOVA)

SECOND PHASE

- More accurate analysis only over the most significant factors.
STATISTICAL ANALYSIS

• Significant:
  a) Film   b) surface c) temperature
STATISTICAL ANALYSIS

- Not significant:
  - d) Speed
  - e) Interface pressure
IN-DEPTH ANALYSIS

• More accurate analysis only on 3 factors instead of 5
  – $15*4*3*3 = 540$

• Only 588 tests instead of the predicted 4860

• 88% reduction of tests number
ANALYSIS OF VARIANCE
ONE FACTOR DESIGN
STATISTICAL MODEL

Y = dependent variable
- Result of the test
- Answer variable (guiding parameter, sales volume)
- Quality indicator (characteristic value of a product)

X = independent variable
- Potential influential factor
STATISTICAL MODEL

• Final aim of the experiment: to investigate the bound between X and Y ↔ **impact of X over Y**

**Mathematical model:**

\[ Y = f(X, \varepsilon), \varepsilon \text{ represents the impact of all the other factors except } X \]

**Es:**

\[ Y = \text{lifetime of a battery}, \ X = \text{brand}, \ \varepsilon = \text{other factors not completely known but determining the “experimental uncertainty” (e.g. Environmental factors)} \]
### Statistical Model

- **Levels (different brands)**

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>……</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$Y_{11}$</td>
<td>$Y_{12}$</td>
<td>……</td>
<td>$Y_{1C}$</td>
</tr>
<tr>
<td>2</td>
<td>$Y_{21}$</td>
<td>$Y_{22}$</td>
<td>……</td>
<td>$Y_{2C}$</td>
</tr>
<tr>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
</tr>
<tr>
<td>R</td>
<td>$Y_{R1}$</td>
<td>$Y_{R2}$</td>
<td>……</td>
<td>$Y_{RC}$</td>
</tr>
</tbody>
</table>

- $i = 1, 2, ..., R$ 
- $j = 1, 2, ..., C$

Level

Repetition

\[
Y_{ij} = \mu + \tau_j + \varepsilon_{ij}
\]
STATISTICAL MODEL

\[ Y_{ij} = \mu + \tau_j + \varepsilon_{ij} \]

\( \mu \) = Global true mean, over infinite tests ("grand mean")

\( \tau_j \) = true differential effect associated to \( j \)-th level of the factor (how much the result of each level is different from the grand mean)

\( \varepsilon_{ij} \) = noise, error associated to the (i,j)-th value
STATISTICAL MODEL

- \( Y_{ij} = \mu + \tau_j + \varepsilon_{ij} \)
- By definition the sum of deviations from the grand mean is equal to 0:
  \[
  \sum_{j=1}^{C} \tau_j = 0
  \]
- The experiment produces \( R \cdot C \) results \( Y_{ij} \)
- The analysis allows to estimate (but not calculate: it would require infinite tests) \( \mu, \tau_1, \tau_2, ..., \tau_C \), but not \( \varepsilon_{ij} \). These can be estimated only indirectly by subtraction.
STATISTICAL MODEL

- $\bar{Y}_j$ indicates the mean per column: it is the average of the R repetitions per each level (one battery brand)

- Intuitively: different $\bar{Y}_j \rightarrow$ different results per each brand (X), therefore the brand (X) might have an impact on the battery life (Y)
STATISTICAL MODEL

- $\bar{Y}_{..}$ is the "grand mean"

$$
\bar{Y}_{..} = \frac{\sum_{j=1}^{C} \bar{Y}_{..j}}{C} = \frac{\sum_{j=1}^{C} \left( \sum_{i=1}^{R} Y_{ij} \right)}{RC}
$$
STATISTICAL MODEL

• $Y_{ij} = \mu + \tau_j + \varepsilon_{ij}$

• $\bar{Y}..$ Represents an estimation of $\mu$, global mean of the random variable

• $(\bar{Y}.j - \bar{Y}..)$ represents the estimation of $\tau_j$, gap of each level from the mean
STATISTICAL MODEL

\[ Y_{ij} = \mu + \tau_j + \varepsilon_{ij} \]
Substituting:
\[ Y_{ij} = \bar{Y}_{..} + (\bar{Y}_{.j} - \bar{Y}_{..}) + \varepsilon_{ij} \]
therefore:
\[ \varepsilon_{ij} = Y_{ij} - \bar{Y}_{.j} \]
As we previously introduced, it is not possible to identify an analytical formula for the estimation of the error: it can only be indirectly determined by difference.
Therefore the following:

\[ Y_{ij} = \mu + \tau_j + \varepsilon_{ij} \]

Can be written as:

\[ Y_{ij} = \overline{Y}.. + (\overline{Y}.j - \overline{Y}..) + (Y_{ij} - \overline{Y}.j), \] from which:

\[ Y_{ij} - \overline{Y}.. = (\overline{Y}.j - \overline{Y}..) + (Y_{ij} - \overline{Y}.j) \]
STATISTICAL MODEL

\[ Y_{ij} - \bar{Y}_{..} = (\bar{Y}_{.j} - \bar{Y}_{..}) + (Y_{ij} - \bar{Y}_{.j}) \]

Total variation in \( Y \)  

Variation of \( Y \) associated to the levels (column), therefore associated to \( X \)  

Variation of \( Y \) associated to other factors (error)

Not related to \( X \), because \( j \) is always the same: it represents the variability among repetitions
SUM OF SQUARES

\[ Y_{ij} - \bar{Y}_{..} = (\bar{Y}_{.j} - \bar{Y}_{..}) + (Y_{ij} - \bar{Y}_{.j}) \]

Squaring both members and summing in i and j:

\[
\sum_{j=1}^{C} \sum_{i=1}^{R} (Y_{ij} - \bar{Y}_{..})^2 = R \cdot \sum_{j=1}^{C} (\bar{Y}_{.j} - \bar{Y}_{..})^2 + \sum_{j=1}^{C} \sum_{i=1}^{R} (Y_{ij} - \bar{Y}_{.j})^2
\]

- **TSS**: Total Sum of Squares
- **SSB\(_C\)**: Sum of Squares Between Columns
- **SSW\(_C\)**: Sum of Squares Within Columns
SUM OF SQUARES

\[ SSB_C = R \cdot \sum_{j=1}^{C} \left( \bar{Y}_j - \bar{Y}_{..} \right)^2 \]

- Variance related to the variability among the columns (levels of the factor) and therefore to the impact of the variable X over Y
- \( R \): Amplifier coefficient. It is equal to the number of rows, that is the number of repetition used for the tests of each level. The \( SSB_C \) is greater the higher is the value of \( R \), that is, the more the statistical sample is significant. A result is as important as higher is the number of repetitions used to obtain it.
- Suppose to obtain the brand C (var. X) has an average life time (var. Y) of 120 h, 20% higher compared to the global mean of 100h. This result is less relevant if we tested only 2 batteries per each brand (sample size not significant!) but it is much more important if we tested 200 batteries.
SUM OF SQUARES

\[ SSW_C = \sum_{j=1}^{C} \sum_{i=1}^{R} (Y_{ij} - \bar{Y}_j)^2 \]

- Variance related to the influence of other factors except X. This are usual environmental factors, like temperature or humidity and, in their variations, they build up the kind of noise usually called “experimental uncertainty”. This observation bind the experimental error to the SSW and for this reason it is often called SSE, where E is “Error”.

- If, for instance, the value of SSW\(_C\) is very similar to TSS and SSB\(_C\) is almost zero, than this is an index that the factor X cannot explain the variability of Y: its influence is lost within the experimental uncertainty. Vice versa, if SSB\(_C\) is almost the same as TSS, than the effect of X is evident over any uncertainty.
## SUM OF SQUARES

<table>
<thead>
<tr>
<th>Variance</th>
<th>SSQ</th>
<th>DoF</th>
<th>MSQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between columns</td>
<td>$SSB_C$</td>
<td>(C-1)</td>
<td>$\frac{SSB_C}{C , , - , 1}$</td>
</tr>
<tr>
<td>Within columns (error)</td>
<td>$SSW_C$</td>
<td>C(R-1)</td>
<td>$\frac{SSW_C}{C , (R , - , 1)}$</td>
</tr>
</tbody>
</table>
ONE FACTOR ANOVA

• Why must we divide by the degrees of freedom (DoF) and what are they?
• It is required to divide by the DoF in order to have comparable terms. The DoF are equal to the number of terms in the sum minus one. DoF identify the terms “free to change”.
• Suppose to have a group of 21 persons, including me. If I say that the average age is 28 years, clearly I say nothing about my age. This parameter is free to change. On the other hand, if each declare his age we have 20 new data in the problem and everybody can obtain my age by difference. Therefore, among the 21 values of age, only 20 (21-1) are free to change. Consequently, over n values the DoF are n-1. Columns are C, therefore $SSB_C$ has (C-1) DoF. In one column we have (R-1) DoF. $SSW_C$ is computed over each column, therefore $SSW_C$ has C(R-1) DoF. The total DoF are the whole sum of experimental data minus 1 (RC-1).
ONE FACTOR ANOVA

• Example: \( Y = \) battery life (h), \( X = \) brand of the battery

• 8 levels (8 different brands), 3 repetitions (tests repeated 3 times)

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<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>( Y_{ij} )</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>1.8</td>
<td>4.2</td>
<td>8.6</td>
<td>7.0</td>
<td>4.2</td>
<td>4.2</td>
<td>7.8</td>
<td>9.0</td>
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<tr>
<td>2</td>
<td>5.0</td>
<td>5.4</td>
<td>4.6</td>
<td>5.0</td>
<td>7.8</td>
<td>4.2</td>
<td>7.0</td>
<td>7.4</td>
<td></td>
</tr>
<tr>
<td>3</td>
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<td>4.2</td>
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<td>6.6</td>
<td>5.4</td>
<td>9.8</td>
<td>5.8</td>
<td></td>
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### ONE FACTOR ANOVA

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<td>5.4</td>
<td>9.8</td>
<td>5.8</td>
</tr>
</tbody>
</table>

\[
\bar{Y}_j = \begin{array}{cccccccc}
2.6 & 4.6 & 5.8 & 7.0 & 6.2 & 4.6 & 8.2 & 7.4 & 5.8
\end{array}
\]

\[
\bar{Y}_\cdot = \frac{\sum Y_{ij}}{n}
\]

SSB_C = \[3[(2.6-5.8)^2+(4.6-5.8)^2+\ldots+(7.4-5.8)^2]=
\]

=69.12

SSW_C = \[(1.8-2.6)^2+(5.0-2.6)^2+(1.0-2.6)^2+(4.2-4.6)^2+
\]

\[+\ldots+(5.8-7.4)^2]=

=46.72
**ONE FACTOR ANOVA**

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<td>5.4</td>
<td>9.8</td>
<td>5.8</td>
<td></td>
</tr>
<tr>
<td>(\bar{Y}_j)</td>
<td>2.6</td>
<td>4.6</td>
<td>5.8</td>
<td>7.0</td>
<td>6.2</td>
<td>4.6</td>
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<td>5.8</td>
</tr>
<tr>
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<td></td>
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<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
TSS = (1.8 - 5.8)^2 + (5.0 - 5.8)^2 + \ldots + (7.4 - 5.8)^2 + (5.8 - 5.8)^2 = 115.84
\]
### ONE FACTOR ANOVA

<table>
<thead>
<tr>
<th>Variance</th>
<th>SSQ</th>
<th>DoF</th>
<th>MSQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between columns</td>
<td>$SSB_C = 69.12$</td>
<td>$(C-1)=8-1=7$</td>
<td>$69.12/7=9.87$</td>
</tr>
<tr>
<td>Within columns (error)</td>
<td>$SSW_C = 46.72$</td>
<td>$C^<em>(R-1)=8^</em>(3-1)=16$</td>
<td>$46.72/16=2.92$</td>
</tr>
<tr>
<td>Total</td>
<td>115.84</td>
<td>23</td>
<td></td>
</tr>
</tbody>
</table>

*Total data -1*
FISHER COEFFICIENT (F)

- We can show that:
  - \( E(\text{MSW}) = \sigma^2 \), where \( E \) stands for the Expected value, that is the value obtained averaging infinite values.
    - \( \sigma^2 \) = variance (unknown) of the probability distribution of each result. It is supposed constant.
  - \( E(\text{MSB}_C) = \sigma^2 + V_{\text{col}} \), where \( V_{\text{col}} \) represents the variability among the true means of the columns.

\[
V_{\text{COL}} = \frac{R}{C-1} \cdot \sum_{j=1}^{C} (\mu_j - \mu)^2 = \frac{R}{C-1} \cdot \sum_{j=1}^{C} \tau_j^2
\]
**FISHER COEFFICIENT (F)**

\[ MSB_C > 0 \]

Even if the \( \mu_j \) are equal among themselves, the experimental column means are different due to the experimental uncertainty

\[
E(MSB_C) = \sigma^2 + V_{col}
\]

\[
E(MSW) = \sigma^2
\]

The ratio \( E(MSB_C)/E(MSW) \) allow to compare the variability of the columns compared to the error. Since this ratio is unknown we can use some estimations.

\[
F_{calc.} = \frac{MSB_C}{MSW} > 1 \quad \text{Vcol} \neq 0, \text{ difference among the columns (levels), Impact of X over Y}
\]

\[
F_{calc.} = \frac{MSB_C}{MSW} \leq 1 \quad \text{It is not possible to prove that Vcol} \neq 0, \text{ no difference among the columns (levels) is shown, X does not impact over Y}
\]
FISHER COEFFICIENT \((F)\)

- Anyway, even if \(V_{\text{col}}=0\), \(F_{\text{calc}}\) might be \(>1\). \(\text{MSB}_C\) and \(\text{MSW}\) have the same expected value, but that does not mean that they are equal. Therefore, even if the expected value of the ratio is 1, the real value can be \(>\) or \(<1\). We have a 50% chance of having \(\text{MSB}_C > \text{MSW}\) and 50% for the opposite. Consequently 50% of the times \(F_{\text{calc}}\) might be \(>1\) and 50% \(F_{\text{calc}} <1\).

- Example: boys are statistically higher than girls, but if we measure only 10 male students and 10 girls in the national volleyball team it might be that the latter group will be higher than the first.
FISHER COEFFICIENT (F)

Two hypothesis:

$H_0$ (null hypothesis):
All true differences from the mean are equal to 0
$\tau_1 = \tau_2 = \ldots = \tau_C = 0$
Real means all equal
$\mu_1 = \mu_2 = \ldots = \mu_C$
   The only differences are related to the experimental uncertainty

$H_1$ (significant hypothesis):
At least one true difference from the mean is different from 0
At least one mean is different
FISHER COEFFICIENT (F)

Two hypothesis:

• $H_0$: factor X has no influence over Y

$$F_{calc.} = \frac{MSB_C}{MSW} \leq 1$$

• $H_1$: factor X has a significant influence over Y

$$F_{calc.} = \frac{MSB_C}{MSW} \ggg 1$$

• It is not sufficient that:

$$F_{calc.} = \frac{MSB_C}{MSW} > 1$$
FISHER COEFFICIENT (F)

The starting point is always the null hypothesis (like in a trial the hypothesis of innocence): if we do not have sufficient evidence that X influences Y, than we accept $H_0$. Otherwise, in front of an evidence, we reject $H_0$ (and therefore we accept $H_1$).

\[ H_0 \quad H_1 \]

??

STATISTICAL TEST
FISHER DISTRIBUTION (F)

• Fisher probability distribution: is the probability density distribution of $F_{\text{calc}}$
• It exists a “family” of F-distributions, identified by the DoF of numerator (MSB$_C$) and denominator (MSW)
• Domain of the function for $F > 0$
FISHER DISTRIBUTION (F)

- Area below the curve $\rightarrow$ Probability (total area= 1)
- Area of the Tail: probability to obtain what we actually obtained (p-value), starting from the hypothesis that X does not influence Y (null hypothesis: differences due too chance)
- If the probability that these results are only by chance is sufficiently low than we reject $H_0$. 

$P(F_{\text{calc}})$
Probability density

$F_{\text{calc.}}$
FISHER DISTRIBUTION (F)

We reject $H_0$, ($X$ does not influence $Y$) $\rightarrow$ $X$ influences $Y$ ($H_1$).

What does “sufficiently low” means?

Probability threshold (typical value): $\alpha = 0.05$ (significance 5%)

If $p$-value $< \alpha$, we reject $H_0$, if $p$-value $\geq \alpha$, we accept $H_0$.

The value $\alpha = 0.05$ identifies the limit value $c$

Similarly, if $F_{\text{calc.}} \geq c$, we reject $H_0$, if $F_{\text{calc.}} < c$, we accept $H_0$.
FISHER DISTRIBUTION (F)

• **Similarity between the concept of p-value and a jury.**
  • The accused has no alibi, has a car of the same model of the one seen leaving the crime scene, he is physically similar to the murderer, the hat of the victim was found in his car.
  • The jury can judge in two ways.
  • 1) It is not possible that all these clues exist only by chance → The accused is guilty (H₀ rejected)
  • 2) Actually the model and colour of the car are very common, as is the physique of the accused. Accused and victim were friends. The victim can easily have forgotten the hat in the accused car. The probability that these clues are compatible with pure chance is not low → There is no evidence of guilt → acquitted for lack of evidence. (H₀ accepted).
FISHER DISTRIBUTION (F)

• How do we compute the p-value or threshold value of $F_{\text{calc}}$, $c$?

• $p$-value = $f(F_{\text{calc}}, \text{DoF numerator, DoF denominator})$
  - Computable with Excel, function F.DIST
  - `F.DIST (F_{\text{calc}}, \text{DoF numerator, DoF denominator})`

• $c = f(\alpha, \text{DoF numerator, DoF denominator})$
  - Computable using Excel, function F.INV
  - `F.INV(\alpha, \text{DoF numerator, DoF denominator})`
  - $c$ is also computable using tables, using significativity threshold $\alpha$ and DoF.
ANOVA ANALYSIS AND FINAL TEST

- Calculation of p-value and threshold value c.
- Significance level 5% ($\alpha = 0.05$); $\text{MSB}_c = 9.87$;
- $\text{MSW} = 2.92 \rightarrow F_{\text{calc}} = \frac{9.87}{2.92} = 3.38$
- Use of Excel to determine p-value: $\text{F.DIST}(3.38, 7, 16)$
ANOVA ANALYSIS AND FINAL TEST

- Calculation of p-value and threshold value $c$.
- Significance level 5% ($\alpha = 0.05$); MSB$_C$ = 9.87;
- MSW = 2.92 $\rightarrow$ $F_{\text{calc}} = 9.87/2.92 = 3.38$
- Use of Excel to determine $c$: $\text{F.INV}(0.05, 7, 16)$
# ANOVA Analysis and Final Test

<table>
<thead>
<tr>
<th>Variance</th>
<th>SSQ</th>
<th>DoF</th>
<th>MSQ</th>
<th>$F_{\text{calc}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SSB_C$</td>
<td>69.12</td>
<td>7</td>
<td>9.87</td>
<td>$9.87/2.92 = 3.38$</td>
</tr>
<tr>
<td>$SSW_C$</td>
<td>46.72</td>
<td>16</td>
<td>2.92</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>115.84</td>
<td>23</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3.38 > 2.66
We reject $H_0$.
Brand X has an influence over duration Y.
ASSUMPTIONS

• Statistical Model =

\[ Y_{ij} = \mu + \tau_j + \varepsilon_{ij} \]

1. The terms \( \varepsilon_{ij} \) are random and independent variables.
2. The terms \( \varepsilon_{ij} \) are normally distributed and with null expected value.
3. Variance \( \sigma^2(\varepsilon_{ij}) \) constant for each \( i, j \).

Especially the 2\(^{nd}\) and 3\(^{rd}\) hypothesis are quite strong: small variations have negligible effects over the final results.
ASSUMPTIONS

1. Terms $\varepsilon_{ij}$ are random independent variables. Variations from the real mean of the column are independent of each other and not bounded to the real value of the column. One violation might be related to tests performed in different time points.

2. The terms $\varepsilon_{ij}$ are normally distributed and with null expected value.

In order to avoid this restriction it is possible to use non-parametric tests instead of F-test. The most common is the Kruskal-Wallis test.
Questions

Thank you